Ballistic Effects of Transverse Muzzle Motion on Bullet Exit

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Transverse muzzle motion during the exiting process of the rifle bullet causes several effects during the subsequent ballistic flight of that bullet to a distant target. Variations in bullet exit times are generally inversely corelated bullet launch velocities $\mathbf{V_0}$ and introduce variation into their subsequent ballistic flight effects. We are concerned here with transverse barrel motions in the vertical longitudinal plane caused by the recoil of the rifle being fired with its CG directly beneath its bore axis.

The ballistic effects considered here are:

1. Transverse "kick velocity" **ΔV** perpendicular to $\mathbf{V_0}$
2. Variations in Launch Super-Elevation Angle **ϴ**
3. Initial aeroballistic yaw attitude **α** in aircraft pitch up/down direction
4. Initial aeroballistic yaw-rate **α-dot** in aircraft pitching direction.

The transverse kick velocity **ΔV** causes the entire ballistic trajectory to be deflected upward or downward by the angle **δ**, where

$$\boldsymbol{\delta = \mathrm{Tan}^{-1}[\Delta V/V_0]}$$

This deflection angle is normally in the vertical plane and is caused by recoil induced transverse shear-wave barrel vibrations in that vertical plane. The kick velocity **ΔV** is the transverse velocity of the muzzle at the time of bullet exit for each shot. **ΔV** is on the order of tens of millimeters per second, while $\mathbf{V_0}$ is on the order of hundreds of meters per second, so the trajectory deflection angle **δ** is on the order of **0.10 milliradians**.

An available Excel spreadsheet allows calculation of muzzle motions at the time of bullet exit. It requires specification of the rifle barrel dimensions and material properties as well as interior ballistics program calculations of the cartridge's base pressure curve and the time of bullet exit. We rely upon QuickLOAD© software for these interior ballistics calculations. By adjusting the barrel vibrations using tuning masses attached to the muzzle, we can tune the barrel for bullet exit at a vibrational reversal time when the muzzle is transversely stationary in inertial coordinates. Such tuning minimizes **ΔV** and its angular trajectory deflection effect **δ**.

Variations in launch super-elevation angle **Ɵ** with variations in bullet exit time also angularly deflect the entire ballistic trajectory in flat firing. This effect is primarily caused by recoil induced vertical plane transverse barrel vibrations rotating the last few inches of the rifle barrel about a node located approximately **20 percent** of the barrel length **L** behind the muzzle. Without any extra mass attached to the muzzle, the outer few inches in length of a plain rifle barrel remains straighter than do other portions of the barrel while undergoing transverse shear-wave vibrations in a vertical plane. For some rifles there might also be an upward rebounding away from a solid fore-end or barrel support as well as a “Bourdon tube” straightening of the gravity-drooped barrel caused by pressure inside the bore in addition to tensile-stress straightening caused by rearward recoil dynamics.

For our 338 Lapua Magnum test rifle with its **673 millimeter** (external length **26.5 inches**) barrel tuned for bullet exit at an upward halt (reversal) in muzzle motion, the super-elevation angle **Ɵ** is given by:

$$\Theta = \mathrm{Tan}^{-1}[y(t)/(0.20*L)]$$

$$= \mathrm{Tan}^{-1}[0.021/(0.20*673)]$$

$$= 0.156 \text{ milliradians}$$

where **y(t) = 21.0 micro-meters** at bullet exit.

This is for a free-floated Heavy Varmint profile Schneider barrel having **2.5 pounds** of muzzle brake and extra tuning weights attached to its muzzle. Variations in **Ɵ** from shot to shot are quite small due to our tuning for bullet exit at the time of a reversal in vertical plane muzzle motion. Even these small variations can be further reduced by minimizing the extreme muzzle velocity spreads in group shooting. These test barrels are fitted from Heavy Varmint profile barrel blanks which are fully free-floating in the rifle. The Schneider barrel was custom made with a very fast 7-inch right-hand twist rate for its P5 rifling for optimum firing of long monolithic copper Ultra-Low-Drag (ULD) rifle bullets. Extreme muzzle velocity spreads are brought to low single digits by suing precision reloading techniques and by basedrilling these solid copper prototype bullets using a **0.152-inch (0.45 caliber)** drill diameter. By ducting the base pressure inside the rear driving and gas sealing band, the copper bullets elastically expand enough to seal the peak base pressure driving them down the pressure-expanded bore. No plastic deformation of these fired bullets is observed beyond rifling engravement.

The initial yaw angle **α** is the angle between the bullet's spin-axis direction at launch and its initial velocity vector $\mathbf{V_0}$. Its size is given by the trajectory deflection angle **δ** as formulated above, on the order of **0.10 milliradians**. This aeroballistic yaw attitude **α** might be increased somewhat by passage of the rifle bullet through the highly dynamic muzzle-blast zone before commencing ballistic flight. That being said, this tiny initial yaw angle is basically negligible in subsequent ballistic flight even when firing with an untuned barrel/cartridge combination.

Finally, we come to the rifle bullet's initial aeroballistic yaw-rate **α-dot** as it begins its ballistic flight, which is the primary reason for this paper. Ballisticians use the term "tip-off" to refer to this yaw-rate mechanically imparted by muzzle motion during the time interval of bullet exiting. As a portion of the spin-stabilized bullet's (vector) total angular momentum, this vector initial tumbling rate **α-dot** is carried over into the free ballistic flight of that rifle bullet. For a well made rifle fired correctly, this tip off rate can be considered to be constrained to acting in a vertical plane due to recoil induced standing wave barrel vibrations. The ballistic effects of an initial yaw-rate **α-dot** are difficult to distinguish during post-flight data analysis from those which would have been caused by a much larger initial yaw attitude error **α**. For dynamically stable rifle bullets, either type of initial attitude error damps to insignificance after the first few hundred yards of free flight.

This initial mechanically caused tip-off rate **α-dot** might be increased significantly in magnitude during the bullet's subsequent transit of the muzzle-blast zone before the beginning of ballistic flight. Coning Theory predicts that this initial yaw-rate will produce two significant direct effects in subsequent ballistic flight. One is termed "aerodynamic jump" and is an angular trajectory deflection produced as a transient aerodynamic lift effect while the fired bullet is establishing its initial uniform coning motion, wherein the lift forces rotationally cancel throughout the remainder of the bullet's flight. Indeed, our test firing of these precision made ULD bullets has not produced anticipated accuracy levels even when the muzzle is stationary (**ΔV = 0**) during bullet exit; i.e., better grouping than with un-tuned barrel/load combinations, but still not up to accuracy expectations. We believe that varying aerodynamic jump deflections are producing most of these target impact location errors.

The second ballistic effect of an initial yaw-rate is significantly increased initial aerodynamic "yaw-drag." This shows up as reduced ballistic coefficients (BC's) being measured in early flight. Indeed, using our new Oehler System 89 BC measuring chronograph in our indoor 100-yard test range, we are seeing about 10-percent higher (and more consistent) BC measurements for bullets fired with minimum initial yaw-rate **α-dot**. As an indirect effect of non-zero initial yaw-rate, this increased initial yaw-drag causes increased sensitivity to crosswinds experienced in the first few hundred yards of flight, pushing the bullets farther "downwind" across the target face.

Unfortunately for barrel tuning riflemen, at the time of the typical barrel's reversal of its muzzle motion where **y-dot(t) = 0**, the lateral acceleration at the muzzle **y-double-dot(t)** is typically of ***maximal magnitude***. This is just how the successive time derivatives of the individual mode sinusoidal vibration functions work. For our example tuned 338 barrel, the calculated lateral acceleration at bullet exit is a startlingly large **-266.35 meters per second^2**, or **-27.160 standard "g's"** of **9.80665 m/s^2** each. That is, the halting of the upward muzzle motion requires this much downward acceleration to bring it about. Since our **245.3 grain** copper bullet has a mass of **15.9 grams**, a force of **4.24 Newtons**, or **0.952 pounds**, is being exerted downward on the upper side of the bullet while it is exiting the muzzle. These muzzle accelerations and bullet forces would be even larger for untuned barrels having no massive (**2.5-pound**) muzzle attachments. The large size of these lateral muzzle accelerations is physically caused by the great flexural rigidity of our heavy steel rifle barrel.

> ***As the typical rifle bullet exits the muzzle, it will always lose its attitude guidance within the bore before losing contact with the crown of that bore.***

During the process of exiting the bore, this downward lateral force **F** continuously shifts (linearly in time) from initially having only a purely sideways displacing effect on the attitude-constrained rigid bullet (until the CG exits the muzzle) to finally having a combination of largely rotational effect (pitching the bullet's nose upward) together with some slight remaining downward displacement effect. [Note that we are currently not considering here the dynamically moving mass of the bullet itself inside the barrel in analytically formulating these muzzle motions.]

This tip-off process is quite sensitive to bullet design, especially the location $\mathbf{X_{CG}}$ of the center of mass (or "CG") of the bullet relative to its point of last contact with the crown of the bore. If the CG of the bullet is relatively far in front of its point of last contact, that bullet will acquire relatively more "tip-off" yaw-rate **α-dot** for a given lateral muzzle acceleration. The two extremes in modern rifle bullet designs might be a short, boat-tailed, jacketed lead-core benchrest match bullet (a **65-grain** custom short-range **6mm PPC** bullet) having a large hollow nose cavity (with $\mathbf{X_{CG} \approx 3}$ **to 5 mm**) versus our long-nosed, CNC-turned, monolithic copper ultra-low-drag (ULD) long-range **338 caliber** bullet having a drilled-out base cavity so that $\mathbf{X_{CG} = 13.2}$ **mm**.

Without tumbling, the vector angular momentum **L** of the spinning bullet is given by $\mathbf{I_x * \omega}$ where $\mathbf{I_x}$ is the second moment of inertia of the bullet's mass about its principal spin-axis and **ω** is the spin-rate imparted to the bullet by the rifling. In accordance with the "right-hand rule" convention, this angular momentum vector **L** projects forward (in the direction of the vector spin-rate **ω)** along the spin-axis for our right-hand spin-stabilized rifle bullet.

An impulsive torque $\mathbf{\Gamma_{IMP}}$ is applied at the rear point of last contact of our example bullet during its exit from the muzzle by the downward force **F**:

$$\mathbf{F = m*A = -4.24\ kg\text{-}m/s^2}$$

where $\mathbf{m}$ **= mass of the bullet = 0.0159 kg**

$\mathbf{A}$ **= lateral acceleration of the bullet =** $\mathbf{-266.35\ m/s^2}$**.**

And, the impulsive torque $\mathbf{\Gamma_{IMP}}$ is given by

$$\mathbf{\Gamma_{IMP} = 0.5*F*X_{CG}*\Delta t = -4.2123 \times 10^{-7}\ kg\text{-}m^2/s}$$

where $\mathbf{X_{CG} = 0.0132\ m}$

$$\mathbf{V_0 = 876\ m/s}$$

and $$\mathbf{\Delta t = X_{CG}/V_0 = 15.068\ \mu\text{-sec}}.$$

The vector rate of change **dL/dt** in angular momentum **L** of the flying bullet is equal to the negative of the torque impulse $\mathbf{\Gamma_{IMP}}$ here, as it affects the nose of the bullet. This incremental change vector **dL** lies in the plane perpendicular to **L**.

The effect of the torque impulse $\mathbf{\Gamma_{IMP}}$ on the nose of the rifle bullet is also given by

$$\mathbf{\Gamma_{IMP} = -I_y * \alpha\text{-dot} * 1_{up}}$$

where $\mathbf{I_y}$ is the bullet's transverse second moment of inertia and $\mathbf{1_{up}}$ is a vertical unit vector in the nose-up direction at a right angle to the vector **L**. The second moment of inertia $\mathbf{I_y}$ can also be expressed as $\mathbf{m * k_y^2}$, where $\mathbf{k_y}$ is the radius of gyration of the bullet's mass distribution about a lateral principal axis through its CG.

So, substituting and collecting terms:

$$\mathbf{\Gamma_{IMP} = -I_y * \alpha\text{-dot} = m * k_y^2 * \alpha\text{-dot}}$$

$$\mathbf{= F * X_{CG} * \Delta t = m * A * X_{CG}^2/(2 * V_0)}$$

or $$\mathbf{\alpha\text{-dot} = -0.5 * (A/V_0) * (X_{CG}/k_y)^2}$$

For our example here,

$$\mathbf{A = -266.35\ m/sec^2}$$

$$\mathbf{m = 0.01590\ kg}$$

$$\mathbf{X_{CG} = 0.0132\ m}$$

$$\mathbf{V_0 = 876\ m/sec}$$

and, from solid modeling of the monolithic copper ULD bullet,

$$\mathbf{I_y = 43.381\ grain\text{-}inches^2 = 1.81357 \times 10^{-6}\ kg\text{-}m^2}$$

$$\mathbf{k_y = 0.010679\ m}$$

So, **α-dot = 0.2323 radians/sec. = 13.31 degrees/sec.**

While small, this mechanically imparted initial yaw-rate **α-dot** is not completely insignificant. This yaw-rate **α-dot** likely will be substantially increased in size by passage of the rifle bullet through the muzzle-blast zone immediately before it commences ballistic flight. However, this reversed aerodynamic yaw-rate increase likely will be inconsistent from one shot to the next, as indicated by the large variances often observed in BC measurements.

The initial yaw-rate at the beginning of free flight has significant impact upon subsequent aeroballistic behavior of the bullet. As the bullet's nose is pitching vertically upward in this example, the commencement of right-hand coning motion in early ballistic flight is made easier for a right-hand spinning bullet fired through a right-to-left crosswind in front of the muzzle. The bullet settles into uniform clockwise coning motion after the first half coning cycle. The initial motion of the spin-axis direction is the vector sum of the upward initial yaw-rate and a "downwind" (right-to-left) initial movement due to the right-to-left crosswind. The spin-axis direction then circles more upward into steady clockwise coning motion about the eye of the apparent wind approaching from the right side of the bullet. The initial coning angle **α**, established after a half coning cycle might be slightly smaller than it would have been with no initial upward yaw-rate, all else being equal.

However, had the crosswind been left-to-right instead, this same upward initial yaw-rate **α-dot** would result in a much larger initial coning angle **α**. This is because that initial upward yaw-rate **α-dot** has to be overcome before the spin-axis can spiral downward to begin uniform clockwise (right-handed) coning motion about the leftward displaced eye of the apparent wind. The spin-axis of the bullet first has to circle out rightward before it can move back to its left and begin its steady coning motion. This non-zero initial yaw-rate **α-dot** creates a differing ballistic behavior depending upon the sense of the rifling twist (LH or RH) and the direction from which the crosswind approaches the firing point. Perfect symmetry in left/right crosswind responses requires ***zero*** initial yaw-rate **α-dot**.

The larger initial coning angle **α** in an unfavored initial yaw-rate situation causes an additional yaw-drag penalty, proportional to the square of the effective yaw error **α**, and this additional air drag causes more windage correction to be needed to compensate for downrange crosswinds. This non-zero initial pitching upward or downward yaw-rate **α-dot** explains these crosswind ballistic differences observed by David Tubb and other precision riflemen.

It now becomes apparent that barrel/cartridge tuning for bullet exit during a lateral halt in muzzle motions might not be sufficient for firing certain types of rifle bullets, especially our example long copper ULD bullet used here. Fortunately, the available spreadsheet can show us how to tune for near zero vertical muzzle acceleration where muzzle velocity **ΔV** might be significant, but remains locally invariant over nearby exit times. Ideally, a barrel tuning combination might be found whereby both lateral muzzle velocity and lateral muzzle acceleration are minimized (simultaneously) during bullet exit.

The spreadsheet calculations indicate that one way we can achieve this "zero-force" tuning would be by shortening the barrel by **1.74 inch**, to **26.00 inches** OAL, and attaching a lightweight **3.5 ounce** muzzle brake (Barrett MRAD MB threaded 0.75"x24 TPI) with its CG located **0.65 inch** ahead of the new muzzle crown. The new bullet exit time is indicated by QL to be **1306 microseconds** at **857 meters per second** when muzzle cross-track acceleration is **-2.23 (essentially zero) meters per second squared**. The muzzle position at bullet exit **y(t)** is **36.8 micro-meters** above its neutral position, and the muzzle is moving upward (**ΔV**) at **56.5 millimeters per second**.

At this tuning point,

$$\boldsymbol{\delta = \alpha_0 = \mathrm{Tan}^{-1}[\Delta V/V_0] = 0.0565/862 = 0.066 \text{ milliradians}}$$

and

$$\boldsymbol{\Theta = \mathrm{Tan}^{-1}[y(t)/(0.20*Lext)] = 0.293 \text{ milliradians.}}$$

So, the gross shift in point of bullet impact (**ϴ + δ**) is **0.359 mrad** upward.

Further spreadsheet studies indicate that by mounting the rearmost **8 inches** of a cylindrical portion of this barrel by clamping into an **8-inch long** rigid barrel-block, an oddly better tuning point can be achieved. This **25.5-inch** barrel tapers from **1.250 inches** OD at the front of the barrel-block down to **0.900 inch** OD at its muzzle, over a taper length of just **17.5 inches**. Here, we are treating the front face of the clamped barrel-block as if it were the receiver face for a free-floated rifle barrel.

By threading the muzzle **0.75x24 TPI**, and attaching an efficient muzzle brake and a tuning mass totaling **23.35 ounces** in combined weight, the bullet exits at **1306 micro-seconds** with a lateral acceleration of just **0.210 m/sec^2**. The interesting thing about this tuning is that the locally invariant lateral velocity **ΔV** of the muzzle is just **8.348 mm/sec**, which is quite small.

Building rifles which facilitate this type of “super-tuning,” achieving the best of both types of tuning discussed above, might allow the firing of any type of rifle bullet to its full performance potential.