BARREL HARMONICS ASSOCIATED

CHARGE WEIGHT TUNING FOR ACCURACY

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INTRODUCTION

It is well established that barrels vibrate with a sinusoidal motion, including a multitude of frequencies and amplitudes for various nodal patterns (Reference 1). Optimizing the charge weight to manipulate bullet exit timing vs these vibrations is an accepted method of achieving accuracy, but the specific frequencies and location on the sine wave are often times the subject of debate due to our lack of knowledge regarding the actual behavior. For example, some contend maximum accuracy occurs at the peak or valley of the sinusoidal vibration where barrel movement is momentarily "stopped" while others suggest the mechanism of positive compensation whereby faster bullets exit at a lower muzzle position vs the slower ones.

Instead of shooting a multitude of shots at various charge weights and judging group size to discern the optimum, Creighton Audette proposed simply shooting a single shot and selecting the charge range where the movement in vertical point of impact (POI) was minimal to identify the "node", and at this point theorized the group size would be minimized. This method and several variants of it (e.g. two shot ladder, OCW, etc.) are widely used for load development, especially by long range shooters where velocity differences cause a major detriment on vertical POI dispersion. Using principles presented by Kolbe (Reference 2) and Vaughn (Reference 3) a methodology has been developed to easily analyze target results from an Audette ladder type test in order to characterize the barrel frequency and amplitude associated with this point of impact node.

At this stage the dominant low frequency transverse bending barrel vibrations which are often presented via engineering calculations and associated simulated videos (egg Vaughn and Varmint Al) do not appear to play a role with respect to charge weight tuning, which appears to be most related to longitudinal force transmission at the much higher speed of sound frequencies as proposed by Long, but due to a different mechanism.

These results can be applied to improve one's perspective of load development, and utilized as a simple characterization tool to help understand how various barrel and loading parameters affect the harmonics which are actually involved on the target. For example, it appears that heavier bullets generally result in a larger vibration amplitude. And since the poi node is associated with positive compensation, the optimum velocity required at shorter distance is not the same as for longer distances and result in different moa sensitivity.

BACKGROUND

Vaughn and Kolbe carried out a number of evaluations whereby a rifle was instrumented in a manner to measure movement of the muzzle during shooting and the subsequent effect on accuracy. These and numerous other vibration studies focus on transverse bending wave behavior whereby the barrel/muzzle vibrates up and down like a ruler suspended on the edge of a table does after it is struck to initiate movement. While providing understanding, the associated calculations of vibration frequencies and harmonics have not provided a direct means to "calculate" a charge weight that would yield an accurate load.

Chris Long (Reference 4) proposed a completely different mechanism based on vibration longitudinally along the length of the barrel, which causes deviations in the diameter/shape of the muzzle. Because this mechanism is a simple function of barrel time (BT – the time the bullet is in the barrel between initiating the shot and its exit at the muzzle), he determined optimum barrel times (OBT) to avoid muzzle diameter fluctuation which is a function of the length of the barrel and the speed of sound in steel. BT can be calculated using Quick Load (QL) and Gordons Reloading Tool (GRT), and compared to the appropriate OBT table to optimize charge weight (CW), seating depth, case capacity, and other factors which affect BT. Many have reported this approach provides charge weights which are very close to optimum accuracy, but few agree fluctuating muzzle diameter is a plausible mechanism.

If the reader is not familiar with the differences between transverse and longitudinal vibration waves, there are many useful videos on YouTube. Varmint Al (Reference 1) presents videos appropriate for barrel vibration based on an engineering simulation program, and the associated frequencies for the various harmonic modes. High speed videos also show this behavior. It is also useful to understand that longitudinal force transmission along the barrel is not instantaneous because it is associated with movement from molecule to molecule, vs bending shapes associated with transverse waves (Reference 5).

In the following sections I will discuss the key aspects of these works as it applies to the development of the "method," and will attempt to utilize the appropriate engineering language for clarity as we in the shooting world take liberties with terminology. **For this discussion the definition of a charge weight node is that charge weight which gives minimum deviation of the vertical point of impact on the target, which results in minimizing the effect of velocity differences.**

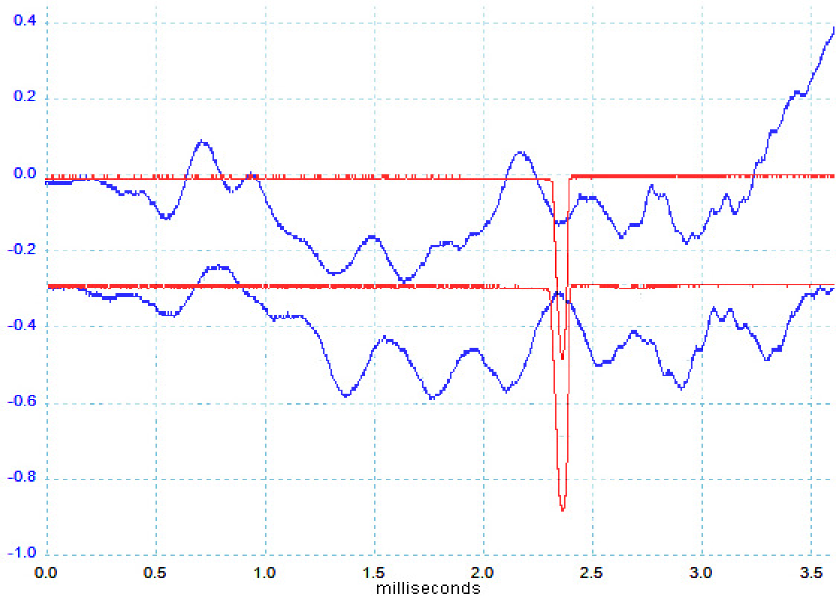
Kolbe (Reference 3)

Dr Kolbe is renowned for his work regarding the physics of shooting, and owned Border Barrels for a number of years.

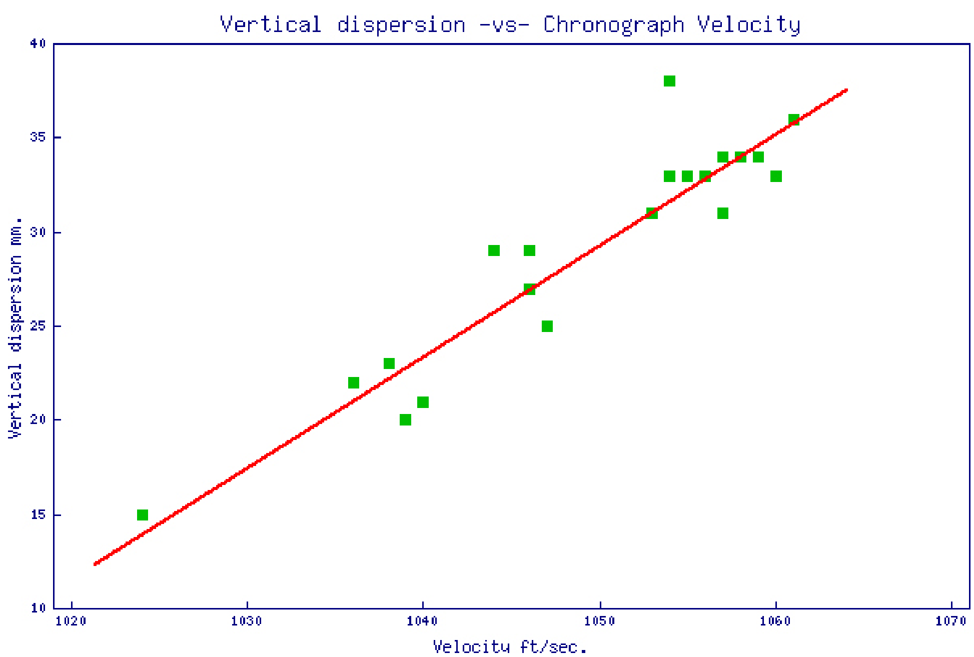
This particular work dealt with the instrumentation of a rimfire rifle, whereby muzzle position was tracked before and after the addition of a tuner. The vertical axis units are voltage, where 0.16V = 1MOA of movement at the muzzle.

In this figure the addition of a 200gm weight (0.44 pound) at the muzzle caused a surprisingly minor extension of the vibration "frequency" or amplitude vs the bare barrel, but a significant **phase shift** between the curves; that is the curve is shifted left to right.

In the image on the right, the top trace is the barrel vibrations from the bare barrel, discussed above, and the bottom trace is the barrel which has now been "tuned" with a 200 gram weight attached to the front of the barrel. The traces have been superimposed so that the bullet exits appear at same time. As can be seen, the general shape or pattern of vibrations for the two traces is very similar. However, while the vibrations on the two traces start out in a very similar way, the pattern of vibrations with the weighted barrel appears slightly stretched in time compared to that of the unweighted barrel.



According to Kolbe "The proposition for **positive compensation** is based on the fact that when any given batch or type of ammunition is chrono graphed, there is always a spread in muzzle velocity observed about a mean. As a consequence, there will be a vertical dispersion in the fall of shot at the target, as the slower bullets in the sample take longer to travel down the range and so drop further than the faster bullets." If the barrel did **not** vibrate the velocity (V) variability and resulting BT would cause the following vertical dispersion as known from basic ballistics.



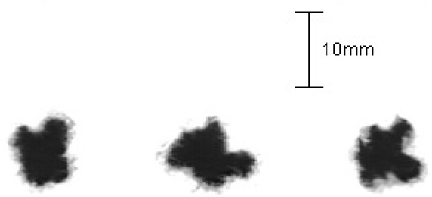
I will refer to this ammo effect of velocity on vertical point of impact as **Ballistic Slope (BS).**

However, using a tuner to time the launch time window such that the faster bullets (lower BT) exit the muzzle when pointed at a lower angle can offset the effect on the target.

I will refer to the vibrational effect of velocity on vertical point of impact as the **Harmonic Slope (HS)-**

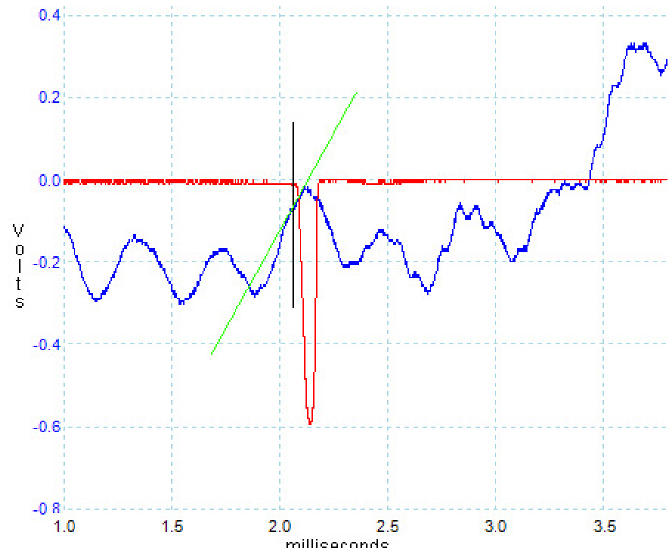
In theory, this barrel with a 200 gram weight on the front should have perfect positive compensation at 50 metres. So, how did it shoot?

See the groups below.



TUNED BARREL GROUPS
(EACH GROUP IS 10 SHOTS AT 50 METRES)

The groups are self evidently 'round', the spread now



“Finally, the barrel was "tuned" by attaching a weight to the front of the barrel, such that the rate of change of angle at the muzzle was now 6.0 MOA per millisecond at bullet launch, which is the rate of change required for **complete positive compensation** at 50 metres. The groups fired with the tuned barrel were small and round, showing no sign of vertical dispersion, so demonstrating that positive compensation had been achieved” according to Kolbe.

I will refer to the degree to which this compensation occurs due to the combined effect as:

Target Slope (TS) = Ballistic Slope (BS) + Harmonic Slope (HS)

And as Kolbe noted the BS can be determined with a ballistic calculator, which represents the external ballistics of bullet flight, and is dependent on the distance and ambient conditions; it is not a constant. For complete positive compensation the harmonic slope must exactly cancel the ballistic slope, across a suitable velocity range such that TS = 0. For a given rifle and bullet combination, there is no guarantee that this can be fully accomplished. The remainder of my investigation will deal with a method to determine the HS using a ladder test on the target.

Also observe that the dominant period of vibration is around 5 cycles in 2 ms = 0.4 ms period = 2.5khz frequency in a 26in barrel.

Vaughn (Reference 2)

While Kolbe focused on rimfire, Vaughn utilized several center-fire rifles for testing. He calculated transverse bending vibrations for a typical barrel as shown below. Mode 1 represents the typical ruler on the table edge behavior, while the other modes demonstrate sinusoidal movement along the barrel length:

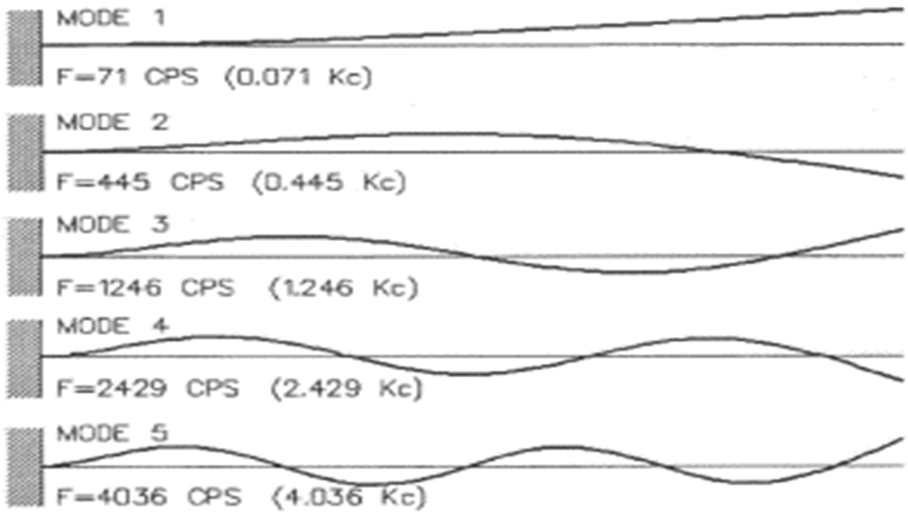


Figure 4-25 - Diagram showing how the barrel vibrates in different modes.

These frequencies are similar to those calculated by Al (Reference 1):

Barrel Harmonics Mode Shape Movies

Mode Movie	Frequency (Hz)	Mode Description
Mode #1	82.0	Cantilever Bending
Mode #2	406	1 Node Bending
Mode #3	1050	2 Node Bending
Mode #4	1756	Torsion
Mode #5	1984	Axial Extension
Mode #6	2485	3 Node Bending
Mode #7	3180	4 Node Bending
Mode #8	4171	1 Node Torsion

Vaughn also used an accelerometer to measure the muzzle displacement, and was very intent on reducing the major impact of the rifle's recoil impulse which occurs in the typical 1.2-1.5 ms barrel time of common centerfires:

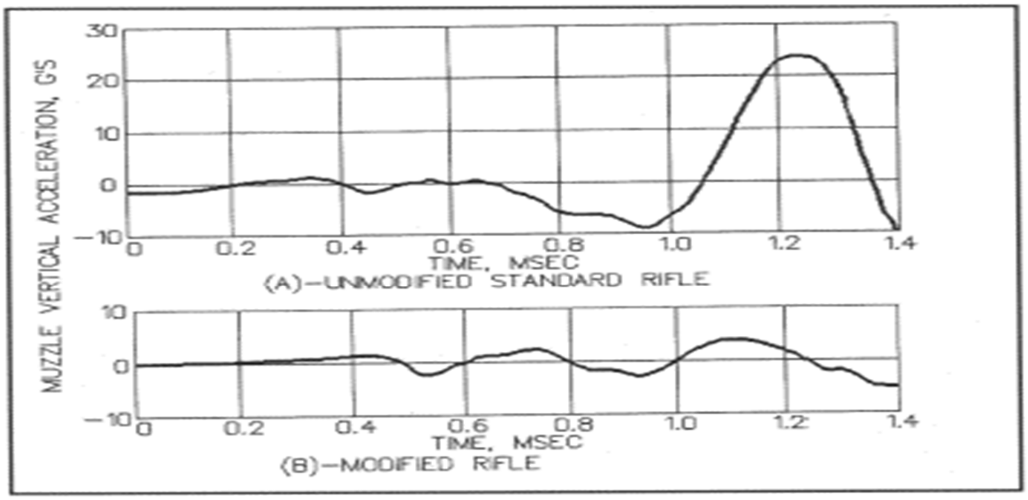


Figure 4-22 - Experimental measurements of muzzle vertical acceleration for the standard rifle (A) with no modifications and the rifle with Recoil Isolator and modified receiver ring (B).

While the major low frequency transverse vibrations and the impact of the recoil impulse can cause accuracy problems, the question arises regarding their importance when optimizing charge weight and loads to achieve optimum accuracy via positive compensation? If these are indeed the primary vibrations affecting accuracy, then these should be discernable on the target!

Interestingly Vaughn also characterized the frequencies observed on the target utilizing a ladder test. The following represent the impact after the ballistic (gravitational) effect has been calculated and subtracted at 100yd (i.e. the ballistic component), and therefore represent the harmonic movement of the barrel. That is, he utilized **HS = BS – TS** to evaluate the individual shots. He reported the smallest group sizes result at the nodes, the inflection points of the sinusoidal point of impact curves observed on the targets, which was Audette's conclusion as well.

CHAPTER 4: BARREL VIBRATION

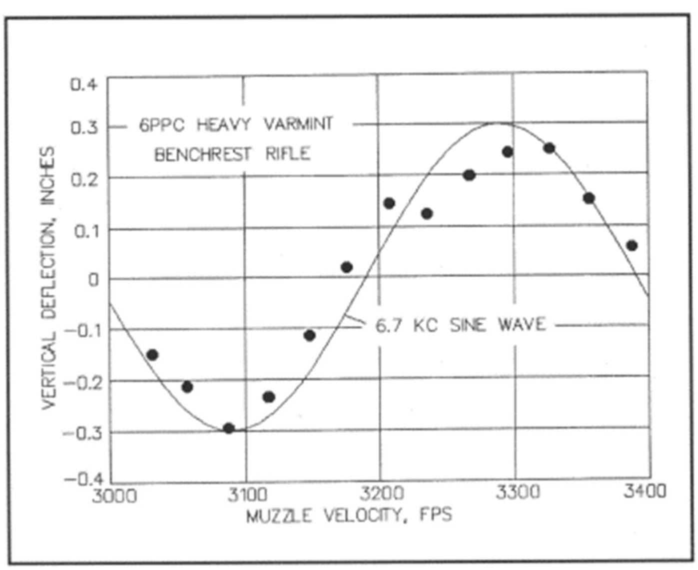


Figure 4-41 - Plot of the vertical positions of group centers for various muzzle velocities on a 6PPC benchrest rifle. The effect of varying gravity drop due to differences in velocity has been removed.

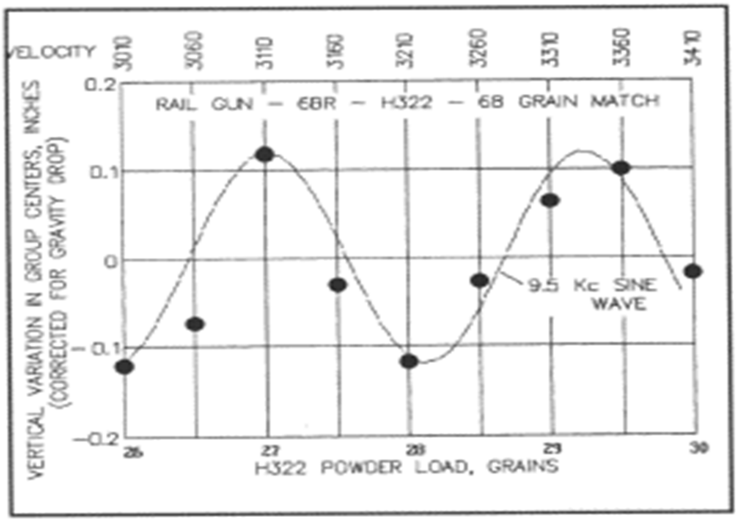


Figure 4-39 - Plot showing variation of vertical position of rail gun center of group impacts at 100 yards at different powder loads and muzzle velocities compared with a 9.5 Kc sine wave. The 9.5 Kc frequency was observed in barrel vibration measurements. The impact points were corrected for differences in gravity drop.

The associated amplitude of the muzzle vibration was reported to be around 0.002 inches, which clearly is not visible to the naked eye. High speed videos commonly exhibit the very large amplitudes associated with the low frequency bending nodes.

Note that these frequencies ranging from 6.7 – 9.5 khz (which was also observed via accelerometer on the muzzle) are not within the frequency range calculated for the transverse bending modes reported by Vaughn and Al, and are much higher/faster. But these high frequencies represent the mechanics which offer the ability to tune and achieve optimum accuracy. So while the primary transverse bending modes and recoil impulse can cause problems, these do not appear to play any role when tuning a load nor do they routinely detract from tuning except when presented as an "outlier" type of problem fault.

Long (Reference 4)

All of the aforementioned references discuss barrel harmonics as a function of transverse vibration, that is the entire barrel/muzzle moves up/down. Or as a ruler vibrates when partially held on a table top and "hit" to excite it, or in other modes such as a rope fixed at one end while the other is swung up / down and the sinusoidal wave is viewed moving along the length. These vibration modes clearly exist as seen in high speed movies.

But as pertaining to load development Long proposed a completely alternative mechanism associated with longitudinal vibration, such as sound waves. He stated

> "Stress Causes Strain or Distortion of the Muzzle – Explaining Observation #2
> What does this stress wave do? Remember that stress is the amount of force or pressure applied to a material, which usually results in the material moving, bending, or displacing. This is called strain. So, the pressure stress from the gasses in the chamber causes a resulting strain in the barrel. Because the stress is applied very rapidly, the some of the stress launches down the barrel as a wave, causing a proportional strain to the barrel as it passes. This strain is initially a slight enlargement of the bore, followed by a slight constriction, eventually dropping off

to no change in the bore diameter at all.

And

As this pulse travels to and fro, it passes by itself, and in the process constructively and destructively adds to itself, all in some predictable way. The shape of the pulse is driven by the pressure/time profile from the propellant burn, and the mechanical properties of the barrel. The theory nicely provides an explanation why very small changes in load parameters could result in large changes in dispersion. If the **muzzle diameter** is changing very rapidly at a particular time after shot initiation, and if the bullet exits at this time, then very small changes in the load will result in small changes in the exit time, but large changes in the exit direction since the muzzle diameter is always different. Think of this as a dynamic variation of the **muzzle crown shape**."

He went on to model this behavior using an engineering simulation program, and generated a table of optimum barrel exit times (OBT) for numerous barrel lengths (BL) such that the muzzle diameter was at a stable, unchanging situation. These OBT conform to the 10%Pmax criteria used by Quick Load to determine BT, and are therefore somewhat less than the actual, total BT. While he does not elaborate on the fundamental criteria used for this simulation it is clear that the nodes are primarily based on the speed of sound through steel (approx. 228 in/ms). This is demonstrated via an excerpt for several OBT nodes and barrel lengths as summarized below, where the speed of the shock wave movement between adjacent nodes is also calculated. Notice the BT difference between nodes 2 and 3 is less than 200 ms, while the difference between nodes 3 and 4 is greater than 300 ms; perhaps this is associated with the constructive/destructive harmonic behavior stated by Long. But across this range of behavior the average speed of the shock wave is 247 in/ms for this example which agrees with a nominal sound wave speed value of 228 in/ms in stainless steel.

BARREL	CHRIS LONG OBT DATA		
LEN	NODE 2vs 3 & 3vs 4		
		MS	DELTA
22	2	0.8235	
	3	0.9386	0.1151
		in/ms	191
22	3	0.9386	
	4	1.0126	0.074
		in/ms	297
28	2	1.0374	
	3	1.1892	0.1518
		in/ms	184
28	3	1.1892	
	4	1.2795	0.0903
		in/ms	310
32	2	1.180	
	3	1.3563	0.1763
		in/ms	182
32	3	1.3563	
	4	1.4573	0.101
		in/ms	317

Using Quick Load and Gordon Reloading Tool many shooters report this OBT criteria often provides a very close starting point for accurate loads based on the target, especially when adjusting the powder burn rate parameter to replicate the actual measured muzzle velocity. Many users and non-users do not accept fluctuating muzzle diameter as the cause, and it seems unlikely. Long's simulated behavior of diameter:

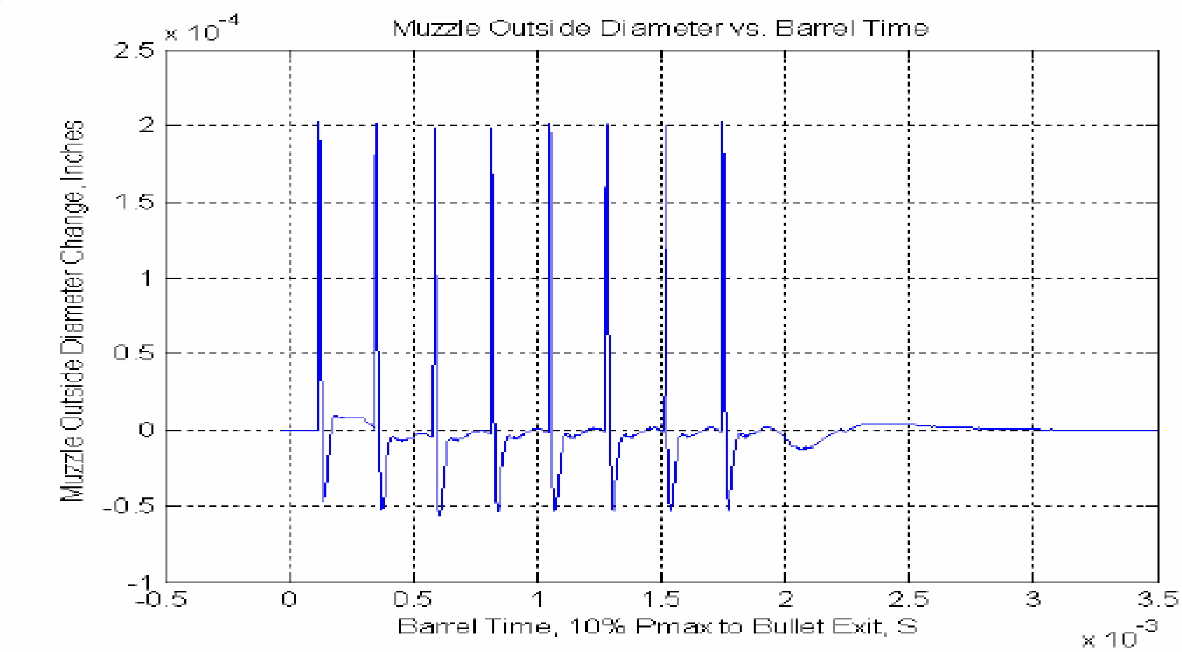


Figure 4 - Muzzle Bore Diameter Change versus Time

This presents a perspective that the diameter is only disturbed for an instantaneous moment during the cycle at which point inaccuracy occurs, unlike the actual observed behavior of an ever-changing sinusoidal motion of the POI on the target as shown by Vaughn. Based on my limited literature research it appears longitudinal energy propagation as proposed by Long can also result in an associated transverse vibration, which would explain the sinusoidal muzzle motion observed on the target which occurs at the speed of sound frequencies. To my knowledge this type of behavior has not been proposed this far, and will be explored further in the next section.

THE METHOD

Using results from a "ladder" test on a target, a method to define the harmonic barrel vibration has been developed using principles demonstrated by Kolbe and Vaughn. This can best be explained using an actual example of a ladder test involving two shots per charge through a 223 using SMK 69 grain bullets.

Based on the vertical POI, there is a charge weight “node” between 24.0-24.5gr of powder.

BARREL HARMONICS CALCULATOR C King 7/1/21

INSTRUCTIONS:

Date:	#####	
Load:	223 69SMK	
Distance:	200	
Barrel Len(in)	26	

(1) Enter the actual results in the yellow highlighted area, the vertical MOA is automatically calculated

(2) The statistical regression of velocity vs charge wt is automatically calculated and determines the velocity used for analysis.

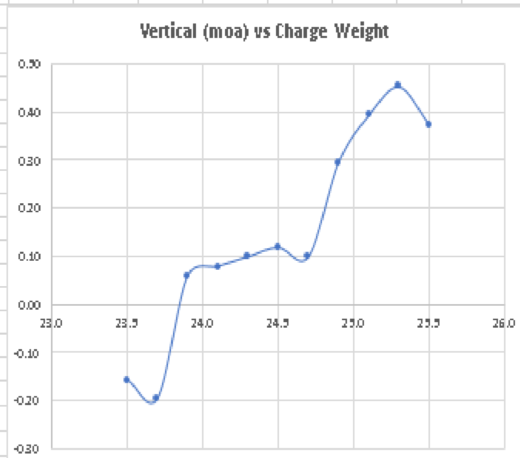
Velocity = 124.3 * ChargeWt + -179

(3) The statistical regression for vertical moa vs velocity is determined as the ballistic trajectory contribution

Bal Vert MOA= 0.0024 * CalcVel + -6.869

(4) The the delta = actual vertical - ballistic trajectory(3) reflects the harmonic behavior

	(1) Actual Results				(2) Calc	(3) Bal	(4) Delta
Gr	Act Vel	Ver(mm)	Vert(moa)		Vel	Vert	MOA
23.5	2740	-8	-0.16		2742	-0.16	0.00
23.7	2778	-10	-0.20		2767	-0.10	-0.10
23.9	2789	3	0.06		2792	-0.04	0.09
24.1	2810	4	0.08		2817	0.03	0.05
24.3	2849	5	0.10		2841	0.09	0.01
24.5	2861	6	0.12		2866	0.15	-0.03
24.7	2898	5	0.10		2891	0.21	-0.11
24.9	2889	15	0.30		2916	0.27	0.03
25.1	2960	20	0.39		2941	0.33	0.06
25.3	2962	23	0.45		2966	0.39	0.06
25.5	2996	19	0.37		2991	0.45	-0.08
24.5	2866	7.5	0.15	AVG	2866	0.15	0.00
0.66	83	10.9	0.21	SD	82	0.20	0.07

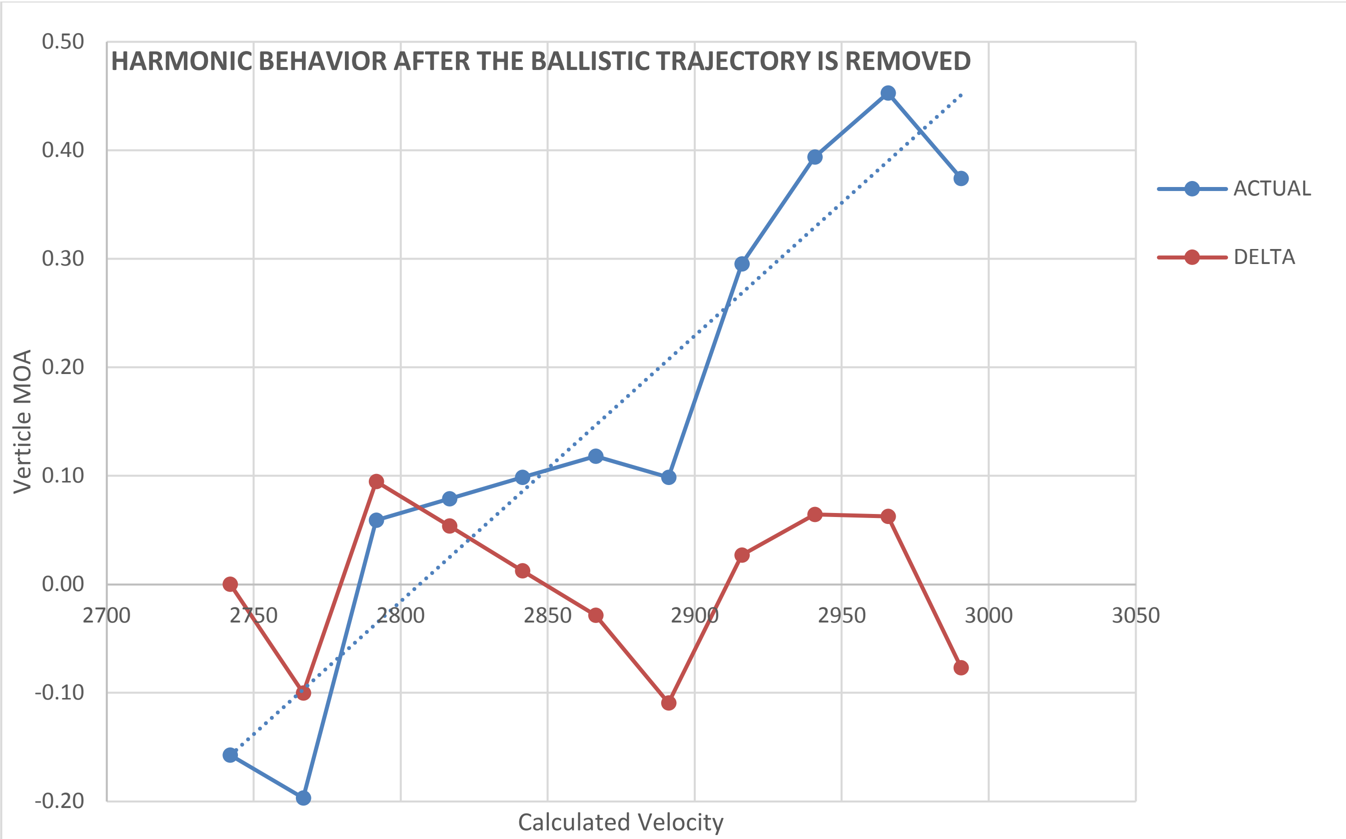


To reduce the effects of measurement noise, the calculated velocity is determined in formula (2) above and used for all future calculations using a linear regression vs charge weight as shown in the formula and table above. Based on the principle of positive compensation as given by Kolbe, the actual POI is the combination of the ballistic slope (BS) plus the effect of the harmonic slope (HS) as **POI = BS + HS** as related to velocity.

To simplify the process and to account for any low frequency vibration, recoil impulse, etc. the BS is determined by simply fitting a linear regression to the POI vs Calc Vel as given in formula (3) above and in the table; this is the dashed blue line below which shows how the POI became higher due to velocity alone.

Now by subtraction the actual POI and the POI due only due to ballistics (using the BS (3) above) represents the harmonic behavior of the barrel, shown below as the “delta” which exemplifies the sinusoidal response as shown by Vaughn. The 2800-2900 fps node corresponds to the 24.0-24.5gr charge node. Note that in this node, the delta POI associated with barrel harmonics demonstrates the barrel is moving down as velocity increases, which exemplifies positive compensation. Due to the ballistics of velocity alone the calculated **BS = 0.0024 moa/fps** and a visual examination of the graph below within the node shows a downward harmonic slope of approximately **HS = - 0.002** such that the resulting target slope is TS = BS +HS = approximately zero. As desired, velocity differences have little to no effect on vertical poi exactly as Kolbe demonstrated!

Using this definition of a node, which is the underlying principle of ladder testing for poi, one can observe how/why the effect of loading irregularities which manifest as velocity differences can be minimized or even negated in order to optimize accuracy.



In this example the node is quite flat and wide because the HS offset the BS "nearly perfectly" due to the combination of the frequency and amplitude of the sinusoidal harmonic curve. But we are not always so fortunate, and the question becomes how to manipulate the frequency and amplitude? In order to do this, we must first be able to characterize these parameters and then learn what features of the barrel, loading components, etc. affect these in order to utilize for our advantage.

Since the deviation in the POI vs the ballistic slope line shown above represents the harmonic behavior of the barrel/muzzle, these values can be fit to a sinusoidal curve to represent the behavior vs velocity. One standard sinusoidal wave form, where the cosine is measured in radians, is:

Vertical(MOA) = Amplitude(MOA) * [Cosine { 2*Pi* (Velocity - Phase Shift) / Period}] + Impact Shift

Again, a number of YouTube videos are available to explain this representation in detail.

To "fit" the results to the curve, the "answers" for amplitude, velocity, period, and phase are input in the red fields below to achieve the best visual fit to the data. For a more precise fit a statistical procedure could be used, but for actual little benefit. In this case a vertical amplitude value of 0.09MOA was chosen as best fit. No adjustment in the Impact Shift was necessary to zero the vertical. You can visually see the actual results repeat the cycle at around 150fps, and a value of 156fps was chosen as the best fit; and in this case no phase shift was necessary to align this periodicity along the velocity axis.

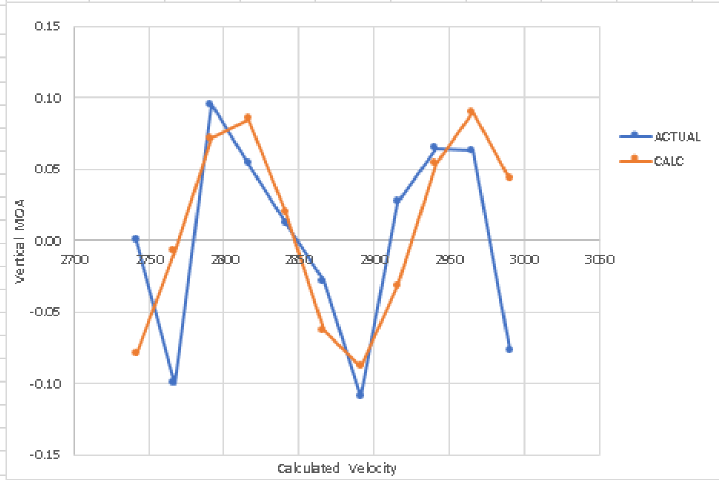
CALCULATE THE HARMONIC RESPONSE

Input Parameters

Amplitude(MOA)	0.09	Impact Shift(MOA)	0
Period(fps)	156	Phase Shift(fps)	0

(2)Calc	(4)Delta		(5)Harmonic	
Velocity	VertActMOA		MOA	Error
2742	0.00		-0.08	-0.08
2767	-0.10		-0.01	0.09
2792	0.09		0.07	-0.02
2817	0.05		0.08	0.03
2841	0.01		0.02	0.01
2866	-0.03		-0.06	-0.03
2891	-0.11		-0.09	0.02
2916	0.03		-0.03	-0.06
2941	0.06		0.05	-0.01
2966	0.06		0.09	0.03
2991	-0.08		0.04	0.12
			AVG	0.01

*Vertical(MOA) = Amplitude(MOA) * [Cosine { 2*Pi* (Velocity - Phase Shift) / Period}] + Impact Shift*

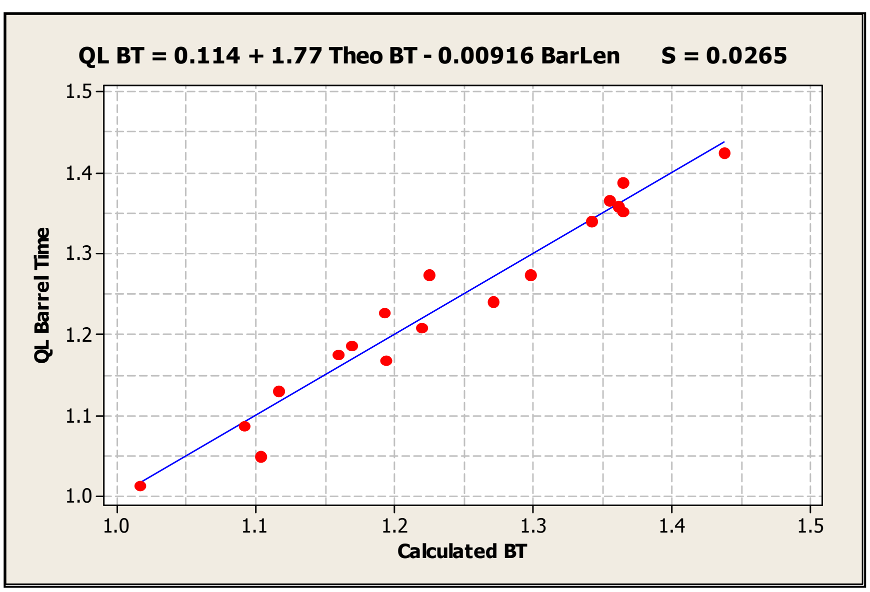


At this point we have quantified the sinusoidal curve which describes the harmonic movement in terms of the initial forcing parameter, velocity (if not measured, velocity can be suitably calculated using QL or GRT). While this form is useful for determining positive compensation based on HS vs BS, velocity is not appropriate for characterizing and understanding barrel vibration and harmonics which are typically referenced to frequency (cycles per second, hz, khz, etc.) as compared to barrel time (BT, ms). But what if you do not have Quick Load to determine BT and use a chrono to measure velocity, which is often used as a parameter to communicate the position of a "node" even though BT is the underlying true determination?

BT can be suitably estimated for this purpose using velocity. An approximate BT can be estimated by:

TheoBT (ms) = Barrel Length(in)/12/Velocity(fps)/1000

This is of course faster than reality since the bullet must accelerate from zero up to the muzzle velocity. Using a number of cases representing a range of barrel lengths, calibers, etc. in Quick Load to determine the BT (based on 10% of Pmax), an excellent correlation was determined by also incorporating barrel length:



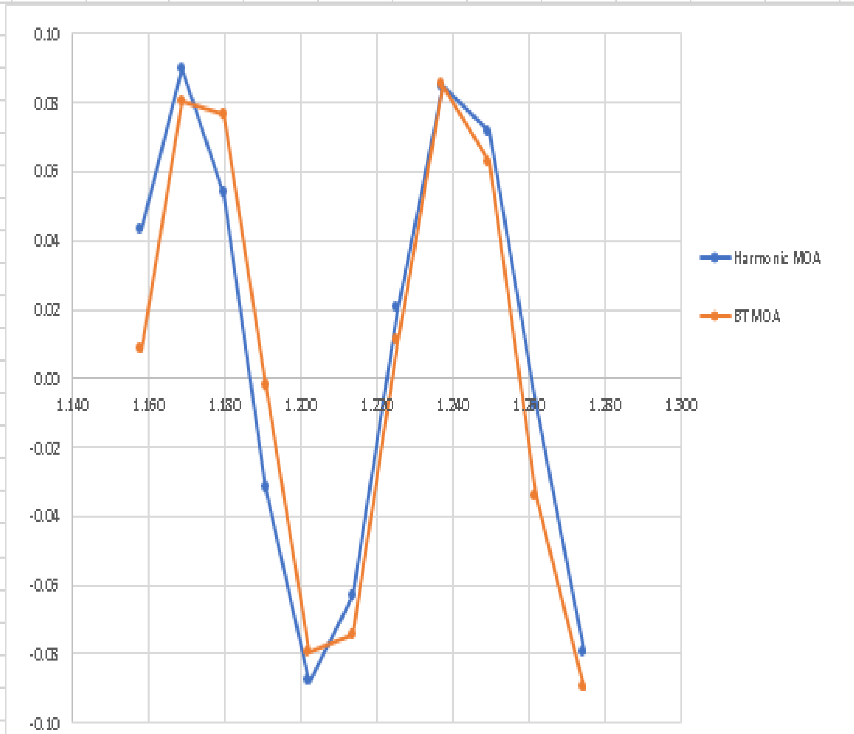
This calculation is efficiently used to convert the velocity results into BT for the purpose of determining the associated time related responses:

Input Parameters

Amplitude(MOA)	0.09	Impact Shift(MOA)	0
Period(ms)	0.0670	Phase Shift(ms)	0.035

Vertical(MOA) = Amplitude(MOA) * [Cosine { 2*Pi* (CalcBT - Phase Shift) / Period}] + Impact Shift

(2)Calc Velocity	Harmonic MOA	Calc BT	BT MOA	BT Error
2742	-0.08	1.274	-0.09	0.01
2767	-0.01	1.262	-0.03	0.03
2792	0.07	1.250	0.06	0.01
2817	0.08	1.237	0.09	0.00
2841	0.02	1.226	0.01	0.01
2866	-0.06	1.214	-0.07	0.01
2891	-0.09	1.202	-0.08	-0.01
2916	-0.03	1.191	0.00	-0.03
2941	0.05	1.180	0.08	-0.02
2966	0.09	1.169	0.08	0.01
2991	0.04	1.158	0.01	0.04
			Avg	0.005



Knowing the exact barrel time is not necessary since it is the differences in the barrel times that are used to determine the frequency, using the methodology previously utilized with velocity. The validity of this methodology was verified using the two examples cited by Vaughn whereby his curves were "digitized" and the cited frequencies of 6.7 and 9.5 khz were confirmed by calculating similar results using this procedure.

The amplitude is the same whether using velocity or BT. Since this is expressed as MOA on the target the corresponding vertical movement of the muzzle is readily determined as:

Muzzle Deflection (thou) = Amp(moa) x Barrel Len(in) / 3.6

and now the frequency of vibration can be calculated simply as:

Freq(khz) = 1 / Period (ms)

In this example for the 223, the muzzle vibration amplitude is 0.7 thousandths of an inch at a frequency of 1/0.067=14.9 khz!

In conclusion, the target results from a ladder test which spans a sufficient charge weight range such that a charge weight node is found can be analyzed to determine the associated barrel vibration frequency and amplitude due to positive compensation.

DISCUSSION

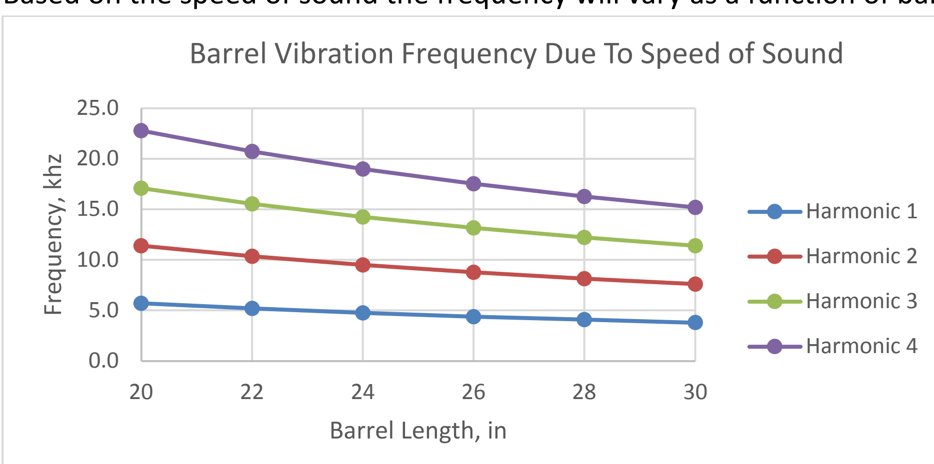
This method has been used to characterize several ladder tests using limited results which I have available, with the following summary:

SUMMARY OF HARMONIC CHARACTERIZATIONS BY RIFLE

SOURCE	CAL	BULLET	BARREL LEN	SHOOTING DISTANCE	BALLISTIC SLOPE	AVERAGE VELOCITY	BT AVG	VELOCITY HARMONICS AMP	PERIOD	PHASE	TIME HARMONICS PERIOD	PHASE	FREQ	BARREL DEFLEC
	(in)	(gr)	(in)	(yd)	(moa/fps)	(fps)	(ms)	A(moa)	(PERfps)	(PHfps)	(PERmS)	(PHmS)	kHz	(thou)
Vaughn 1	243	100	26	100	0	3190	1.078	0.15	200	125	0.086	0.000	11.7	1.1
Vaughn 2	243	100	26	100	0	3190	1.078	0.28	500	300	0.165	0.050	6.1	2.0
DM	270	130	26	450	0.0138	3090	1.117	0.50	200	10	0.085	0.047	11.8	3.6
CK	243	100	26	200	0.0029	2725	1.283	0.32	310	100	0.155	0.115	6.5	2.3
CK	264	140	26	200	0.0012	2835	1.229	0.35	100	90	0.050	0.000	20.0	2.5
CK	264	123	26	200	0.0087	2850	1.221	0.28	162	0	0.068	0.040	14.7	2.0
CK	308	155	26	200	0.0068	2760	1.265	0.53	450	275	0.195	0.010	5.1	3.8
CK	223	69	26	200	0.0024	2865	1.214	0.09	156	0	0.067	0.035	14.9	0.7
CK	223	90	30	200	0.0033	2820	1.408	0.12	149	15	0.075	0.050	13.4	1.0
CK	223	75	30	200	0.0009	2885	1.373	0.35	201	185	0.1007	0.011	9.9	2.9

Key observations:

- All of these frequencies determined from the target (including those reported by Vaughn) are much faster than those associated with the major transverse cycles which are typically calculated based on cantilevered beams (Vaughn, Al, etc.). These periods are in the realm of the OBT node spacing reported by Long, based on the longitudinal transmission of energy at the speed of sound.
- Amplitudes of a few thousands of an inch are in accordance with Vaughn et al.
-
    - Based on the speed of sound the frequency will vary as a function of barrel length:



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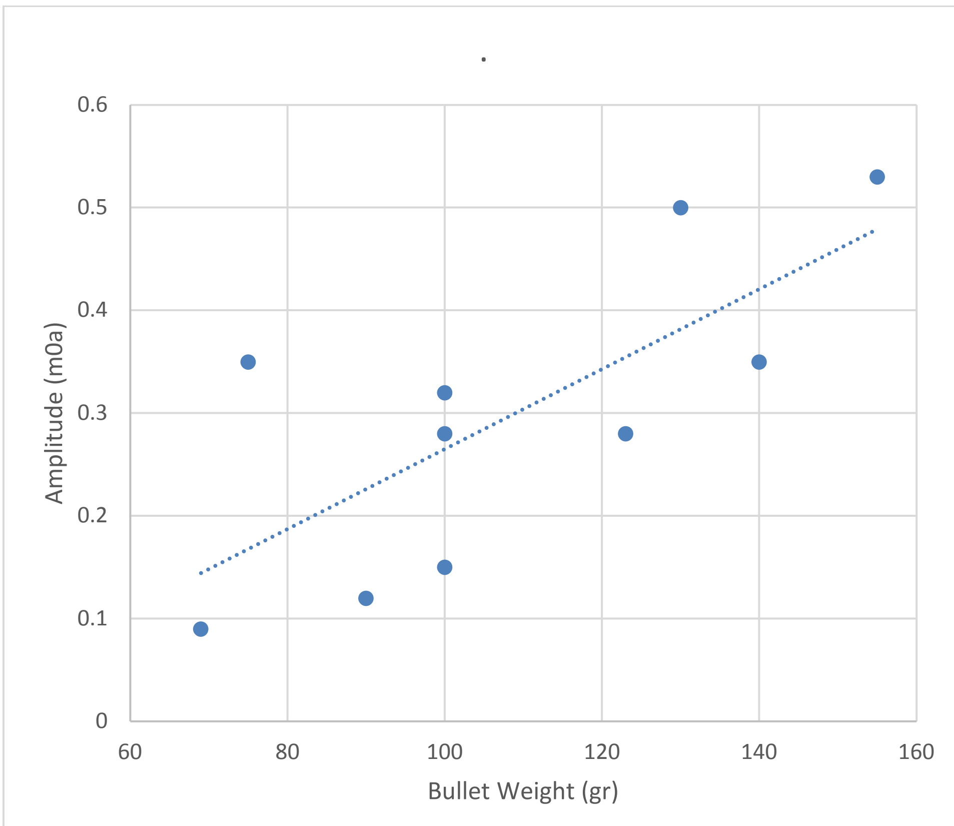
- The speed of sound in steel is the limiting factor which determines these frequencies. It is not a constant, but is a function of the density, elastic modulus, and Poisson ratio (behavior of the barrel in the radial vs longitudinal direction).
- Comparing the observed frequencies in the table, why do some rifles exhibit vibration frequencies which are at faster, higher harmonic nodes while others are at the first harmonic? This is in agreement with those who report target nodes that occur between the OBT nodes.

- Based on this limited data, the correlations of all of the parameters vs each other were examined in an effort to learn what affects the amplitude, etc. Only one interesting correlation was found, which is that of the amplitude of vibration vs bullet weight. Factors such as muzzle energy, etc. were also examined, but this is the only significant response which was found with my limited data set:



Using this characterization method, a much larger data base is required to assess many nuances which can be further speculated:

- How to manipulate the harmonic slope vs the ballistic slope to achieve a wider node utilizing load components, as suggested by the bullet weight vs amplitude correlation. Note that the actual harmonic slope is not a constant, but also varies in a similar sinusoidal manner. The "nominal" harmonic slope is given by its height (2 times the amplitude) divided by its duration (½ the period), so learning how various factors affect these present the opportunity to increase the knowledge needed to improve the load tuning process.
-

- Even though the primary vibration frequency is driven at the speed of sound, other barrel design features clearly affect the amplitude. Adjusting a tuner probably functions by altering the phase shift of the vibration vs any amplitude or frequency affect.
-
- While the harmonic slope can be determined at any distance, the degree of positive compensation will vary as the ballistic slope changes at other distances. In this example the ballistic slope is BS = 0.002 MOA/fps at 100yd, while it is 0.041 at 1000yd! Meaning the velocity and charge weight needed to provide the HS necessary at 1000yd is excessive vs 100yd, which can then result in worse MOA at shorter distance.

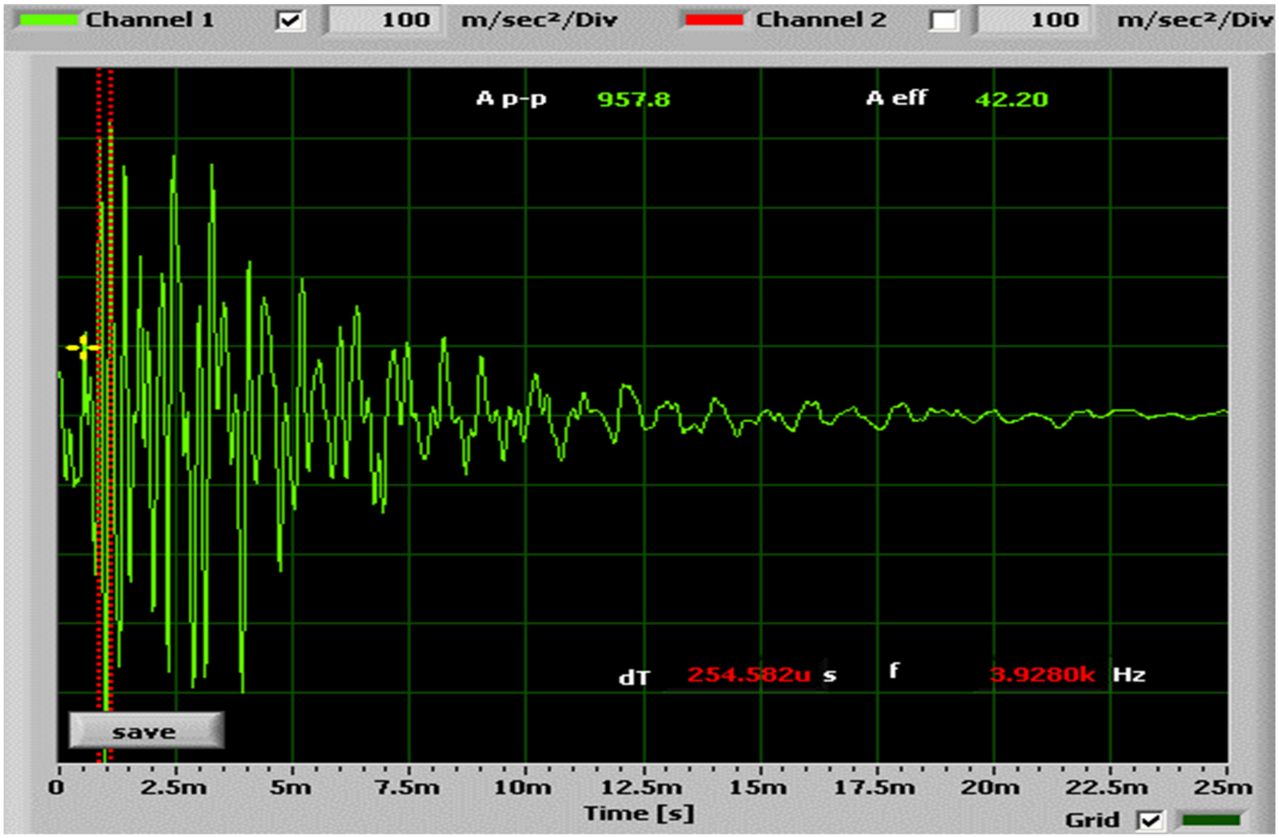
  Thus, the effectiveness and width of the node will vary as well, and the optimum charge weight will also shift based on distance. And at the node how much does the chrono SD drive the vertical POI SD on the target. Mathematically these possibilities can be calculated, but is beyond the scope of this initial article.

Vibration Measurement

While the longitudinal energy/force/shock transmission at the speed of sound best matches the frequencies observed on the target, and is in reasonable agreement with the OBT frequencies, the pattern of muzzle movement is still questionable because the rapid spike-like diameter oscillation proposed by Long does not match the smooth sinusoidal cyclic behavior observed on the target. This strongly suggests there is transverse vertical muzzle vibration which is driven by the longitudinal vibration along the barrel length. To investigate this behavior, I recently procured a high- speed accelerometer to measure muzzle vibration in an effort to “harmonize” my understanding.

A principle of frequency analysis is that the sampling rate must be at least twice as fast as the highest frequency of concern. The Analog Devices ADXL1005Z accelerometer provides a voltage output at 20khz sampling rate for movement in a single direction. This high frequency voltage must be read and converted into a digital signal at a very high rate a well. Surprisingly the sound card in a typical computer offers this capability! The Zeitnitz software offers a means to input the accelerometer voltage output into the laptop microphone input jack, and serve as a high -speed digital oscilloscope digitizing at 41khz.

At this early stage I have only tested by dry firing the rifle to initiate vibration to measure the response at the muzzle. On a 26” barrel the speed of sound will generate a longitudinal vibration of 4.4 khz. This was measured as vertical transverse muzzle vibration at the typical barrel time of 1.2-1.5ms, while at longer barrel times (after the bullet exit) the expected low frequency bending modes emerge:



So, it appears very likely that the longitudinal shock transmission also results in transverse vibration. But much remains to be studied, especially when firing the bullet.

SUMMARY

Using the principle of positive compensation, a method was developed to determine the barrel frequency vibration which is responsible for the target node achieved during charge weight tuning. Interestingly this frequency is not one which is typically associated with transverse bending modes, but results from longitudinal stress propagation at the speed of sound as postulated by Long, but because it results in transverse vibration at the muzzle.

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