Interior Ballistics with Copper Bullets

James A. Boatright

February11, 2019

1.0 Introduction

Monolithic copper bullets do not behave in the rifle barrel as we have come to expect in firing conventional jacketed, lead-cored rifle bullets. Bullet designers, barrel makers, gunsmiths, and riflemen need to understand these differences and limitations to avoid disappointing performance when switching to copper bullets.

We will use the well accepted behavior characteristics in the rifle barrel of premium quality, soft-jacketed, soft-lead-cored, match-type rifle bullets as our benchmarks for interior ballistics performance which we strive to equal or better in firing our monolithic copper rifle bullets.

We will compare the interior ballistics performance characteristics of a new 338-caliber 246-grain monolithic copper Ultra-Low-Drag (ULD) bullet of patented design against those of the conventional 338-caliber 250-grain Sierra MatchKing bullet. The promising new copper bullet is currently well into development and testing.

Our firing tests are done using a single-shot, bolt-action match rifle fitted with various premium quality custom barrels, each using different rifling patterns and rifling twist-rates, but chambered in 338 Lapua Magnum using the same custom chambering reamer.

In particular we shall compare (**1**) the barrel obturation characteristics, (**2**) the shot-start pressures required, (**3**) the bullet/barrel friction characteristics, and (**4**) the bullet spin-up effects of these two example bullets. As a key part of the study of barrel obturation, the maximum internal expansion of typical steel rifle barrels will be calculated for these two different types of 338-caliber bullets.

The interior ballistics performance formulations developed herein should be generally applicable to rifle bullets of different materials, caliber, design, and construction.

2.0 Physical Properties of Materials Used

The materials being studied here are limited to

- ***Essentially Pure Lead***, used as the core material in many benchrest and match-type rifle bullets
- ***Gilding Metal Copper Alloy***, used as the jacket material in these bullets
- ***Essentially Pure Copper***, from which our monolithic bullets are made by CNC machining
- ***Rifle Barrel Steel***, from which our various match-grade rifle barrels are made.

Most military and hunting-style rifle bullets are made with harder core and jacket materials than are typically used when match accuracy and consistency are the main goals. Pure lead is the softest and most dense of the continuum of lead alloys commonly used in making cast or swaged lead bullets and the cores of jacketed bullets. Benchrest rifle bullet cores and many other match bullet cores are usually made of essentially pure lead, and we use it here both as the ideal limiting case and as the one core material for which its material properties are best known. Benchrest bullet makers prefer the softest and most dense lead for precisely pressure-forming (“squirting”) the cores which will subsequently be pressure-seated into gilding metal jackets through their open fronts. Bullet cores made from lead alloyed with a small amount of antimony, such as our example Sierra MatchKing bullet here, are only slightly less dense and slightly tougher and stronger than those made from essentially pure lead.

We are mainly concerned here with the friction characteristics of the gilding metal jacket alloy composed of approximately 95-percent copper and 5-percent zinc. The thin walls of the bullet jacket (**0.032-inch** for our example bullet) cannot contribute much strength in preventing the jacketed bullet from distorting as readily as does its lead core. The jackets may be left in their work-hardened state as formed when making tough military and hunting bullets, but they are usually annealed in some step before the final bullet-pointing operation when making benchrest or open-tipped match-type rifle bullets.

The copper material from which our prototype ULD bullets are made by CNC turning is half-hard, cold-rolled rod stock of UNS C147 99.6-percent Copper with a small amount (up to 0.4 percent) of sulfur added for improved machinability. This half-hard copper state was formerly designated H04, but is now called H02 in the copper industry. The state of hardness of the copper bullet material is critically important in interior ballistics. Annealed copper (H00) is not only much weaker, both in yield strength and in shear strength, but is also notoriously “grabby” in friction against steel.

We will use average or typical physical properties of the many types of carbon-steel alloys, which includes our 400-series stainless-steel alloys, from which either mass-produced rifle barrels can be made by various manufacturing techniques or from which premium quality custom barrel blanks can be made by using either button-rifling or cut-rifling techniques.

The pertinent physical properties of these four materials are tabulated in the table below. Many of these specifications are converted from metric pressure units **MPa** into pounds per square inch (**psi**). One Mega-Pascal (**MPa**) is equivalent to **145.04 psi**. We will also freely use the abbreviation **ksi** which is commonly used in ballistics work to stand for **kilo-pounds per square inch**, or **thousands of psi**.

Materials Properties:	Lead	Gilding Metal	Copper	Barrel Steel	Units
Density	11.34	8.86	8.92	7.85	grams/cc
Weight	2867.80	2240.70	2255.80	1985.20	grains/cu in
Yield Strength (S)	1,740	10,000	40,000	130,000	psi
Modulus of Elasticity (E)	2,030,000	17,000,000	16,900,000	29,000,000	psi
Poisson's Ratio (µ)	0.44	0.33	0.33	0.30	none
Coeff of Friction (Cf)	1.40	0.22	0.36	0.42	Sliding on dry steel

3.0 Physics of Interior Ballistics

An introductory level understanding of the physics of elasticity and strength of materials is necessary to understand how rifle bullets and barrels behave in interior ballistics. Even though the pressures involved are fairly high by everyday standards and the barrel dwell times of rifle bullets are only one to two milliseconds, static and quick-static loading analyses are adequate for our purposes here in which we are primarily concerned with radial and tangential stress effects as opposed to axial-direction effects. We shall use the British Engineering units of feet, inches, pounds, and grains commonly used by American riflemen instead of the SI (international metric) units used professionally.

Much of the numerical data we use comes from interior ballistics simulation runs of the commercially available QuickLOAD© software program. We use QL frequently in load development and recommend it to riflemen. As a principal “reality check,” we compare QL calculations of expected muzzle speeds with measurements made using barrel-attached Magneto-Speed and near-range Oehler 35P chronographs. The 5-yard downrange optical-sensing chronographs usually see about 18 to 20 fps slower bullet speeds than the barrel or muzzle brake mounted inductive-sensing chronographs.

We will develop the elementary concepts and relationships of material strength, stresses and strains, distributed force and pressure, and sliding friction in each section as we need them.

Let us start by looking at the rifling engraved bullet at the moment of peak chamber pressure (**60.0 ksi**, in this example) which occurs when the bullet has travelled just a few inches down the rifle barrel from its initial position. According to the interior ballistics program *QuickLOAD©* the peak base-pressure **P** accelerating our example 246-grain copper 338-caliber bullet is

$$\mathbf{P = 51.0\ ksi}$$

which is **85 percent** of the peak chamber pressure of **60 ksi** for this example 338 Lapua Magnum cartridge and occurs when the bullet has moved just **3.2 inches** from its starting position.

The base-pressure **P(t)** gradually becomes a smaller fraction of the instantaneous chamber pressure (down to **82.5 percent** at the muzzle) as the bullet speed increases and as it travels down the somewhat gas-flow restrictive 338-caliber barrel. Base-pressure **P(t)** is only **10.8 ksi** behind the bullet at the muzzle of our example **28-inch** barrel.

The peak base-pressure **P** exerts a distributed force **F** on the bullet accelerating its entire mass forward. The area $\mathbf{A_B}$ over which this force **F** is distributed can best be thought of as the cross-sectional area of the rearmost barrel- aperture obturating disc, an imaginary plane within the rearmost full diameter portion of the rifle bullet. This plane area $\mathbf{A_B}$ is also ideally equal to the cross-sectional area of the rifled bore of the obturated barrel.

The bullet material in front of this obturating disc is being shoved forward by the distributed force **F(t)**, while the afterbody (boat-tail) material of the monolithic bullet behind this plane of obturation is actually being dragged along via its mechanical attachment to the shank of the bullet.

At any instant of time **t**, we can reason that, hydrostatically,

$$\mathbf{F(t) = P(t)*A_B}$$

Note that, for any given base-pressure **P(t)**, the bullet-driving force **F(t)** increases directly with the cross-sectional area of the bore $\mathbf{A_B}$ and, thus, **F(t)** increases with the square of caliber. This caliber dependence, together with Newton’s Third Law of Motion, basically explains why larger-caliber rifles typically produce more recoil felt by the shooter. Note also that this recoil force **F(t)** does not depend on the mass of the projectile being fired.

That distributed force **F(t)** imposes a peak axial-direction ***stress*** $\boldsymbol{\sigma_a}$ on the copper bullet material within this aperture obturating disc which is given, at the instant of *peak* base-pressure **P**, by

$$\boldsymbol{\sigma_a = F(t)/A_B = P = 51.0\ \mathrm{ksi}}$$

Note that this peak axial stress $\boldsymbol{\sigma_a}$ depends only on the peak base-pressure **P** and is completely independent of both the caliber and mass of the bullet.

If that axial stress $\boldsymbol{\sigma_a}$ does not exceed the elastic limit **S** (**40,000 psi** for “half hard” copper material, or **1,740 psi** for pure lead), it will produce an axial, compressive elastic ***strain ratio*** $\boldsymbol{\varepsilon_a}$ in this thin disc of bullet material given, in accordance with Hooke’s Law, as

$$\boldsymbol{\varepsilon_a = \sigma_a/E = P/E = 0.003018}$$

where **E** is Young's Modulus of Elasticity for our bullet material:

$$\mathbf{E = 16{,}900{,}000\ psi} \qquad \text{(for copper)}$$

and

$$\mathbf{E_L = 2{,}030{,}000\ psi} \qquad \text{(for pure lead).}$$

Young's Modulus of Elasticity is the ratio of stress applied to elastic strain ratio produced for either *compressive* or *tensile* stresses. *Shear* and *bulk* (volumetric) stresses and strains use different elasticity ratios.

Consider just the 338-caliber copper bullet for the moment. We know that this compressive axial stress $\boldsymbol{\sigma_a}$ would, if not constrained by the steel walls of the barrel, produce a radial elastic strain ratio $\boldsymbol{\varepsilon_r}$ in this same copper material within the obturation aperture given by

$$\boldsymbol{\varepsilon_r = \mu^*\sigma_a/E = \mu^*P/E = 0.000996}$$

where $\boldsymbol{\mu}$ is Poisson's Ratio of lateral shrinkage to elongation during tensile and compressive laboratory testing of this copper material:

$$\boldsymbol{\mu = 0.33}$$

At this radial strain ratio $\boldsymbol{\varepsilon_r = (\Delta r)/r}$, the potential radial increase $\boldsymbol{\Delta r}$ in the size of the unconstrained 338-caliber copper bullet would be

$$\mathbf{\Delta r = r^*\varepsilon_r = 0.1693^*0.000996 = 0.000169\ inches.}$$

We can see why this is so if we consider in isolation the thin disc of copper material within the obturation aperture. Unstressed, this thin disc has a radius **r** and a thickness **h**. Under a compressive axial stress $\boldsymbol{\sigma_a}$ within its elastic range, the thickness of this disc is reduced to $\mathbf{h - \Delta h}$, and its unconstrained radius would increase to $\mathbf{r + \Delta r}$.

If the unstressed *volume* of this disc, $\mathbf{A_B{}^*h}$, were to remain *constant* under this axial elastic stress $\boldsymbol{\sigma_a}$, we could say

$$\boldsymbol{\pi^*r^2\ {}^*h = \pi^*(r + \Delta r)^2\ {}^*(h - \Delta h).}$$

Dividing through by $\boldsymbol{\pi^*r^2\ {}^*(h - \Delta h)}$, and neglecting second and higher-order differential terms, this expression simplifies to

$$\mathbf{(\Delta r)/r = 0.5^*(\Delta h)/h}$$

or, in terms of radial and axial elastic strain ratios

$$\boldsymbol{\varepsilon_r = 0.5^*\varepsilon_a}$$

This *constant volume* condition actually holds only for an incompressible liquid such as water in hydrostatics. It is almost statically true for a "perfectly elastic" material such as soft gum rubber which has a value approaching **0.5** for Poisson's Ratio (**μ**). Here, we must replace the value **0.5** with **0.33**, the value of Poisson's Ratio (**μ**) for copper when operating within its elastic range:

$$\varepsilon_r = \mu^*\varepsilon_a = \mu^*\sigma_a/E = \mu^*P/E$$

We now see why Poisson's Ratio **μ** can never exceed **0.5** for any solid material which retains internally some portion of any stress applied to it. The ability of a metal object to retain stress internally also allows it to retain a "memory" of its pre-stressed shape.

Multiplying through by Young's Modulus of Elasticity **E**, we find that the radial stress σ_{rbp} caused by the peak base-pressure **P** acting axially is

$$\sigma_{rbp} = \mu^*\sigma_a = 0.33^*51.0 = 16.8 \text{ ksi}.$$

This is true elastically only for the first **S = 40 ksi** of the peak base-pressure **P** on our copper bullet, but here, since **P > S** even for this stout copper bullet, we must modify this expression slightly. Poisson's Ratio **μ** applies during the initial compression up to the elastic limit **S**, but the remaining overpressure (**P – S**) results in plastic flow which we will treat here as a liquid flow:

$$\sigma_{rbp} = 0.33^*S + 0.5^*(P - S) = \underline{18.7 \text{ ksi}}. \qquad \textbf{(Copper Bullet)}$$

This difference in constrained radial contact pressure due to the peak base-pressure **P** exceeding the elastic limit is even more apparent for our lead-cored bullet since S_L is only **1,740 psi**.

$$\sigma_{rbp} = 0.44^*1.74 + 0.5^*(51.0 - 1.74) = \underline{25.4 \text{ ksi}} \qquad \textbf{(Lead Bullet).}$$

These peak bullet-to-barrel contact pressures σ_{rbp} are due solely to the same peak gas pressure **P** acting on the bases of the two bullets. This is enough information to calculate the internal expansion of the rifle barrel when firing lead our example lead-cored bullet, but is just one part of that calculation for the copper bullet.

As the barrel expands while firing the lead-cored bullet, that bullet just *plastically expands* a little more and maintains nearly the same contact pressure. The peak internal barrel expansion is caused by the sum $\mathbf{P_i}$ of the gas pressure on the base of the bullet **P** and the radial contact pressure $\boldsymbol{\sigma_{rbp}}$ at that bullet base.

$$\mathbf{P_i = P + \sigma_{rbp} = 51.0 + 25.4 = \underline{76.4\ ksi}.}$$

For safety reasons, $\mathbf{P_i}$ must not exceed **100 ksi** in routine firing for many steel rifle barrels, and we would prefer that our new copper bullet never causes greater pressure inside the rifle barrel than does the lead-cored bullet with which riflemen are more familiar.

For all practical purposes when lead or lead-cored bullets are being fired, we can estimate the peak pressure $\mathbf{P_i}$ inside the rifle barrel to be **150-percent** of the peak *chamber pressure* with straight-walled cartridge cases, and between **100-percent** of chamber pressure for the more extremely bottlenecked cartridges and about **140-percent** of chamber pressure when firing lead-cored bullets from moderately bottlenecked cartridges. According to the peak base-pressure calculated in QuickLOAD, $\mathbf{P_i}$ is **127.5-percent** of the peak chamber pressure for this 338 Lapua Magnum load with our example 338-caliber lead-cored Sierra MatchKing bullet.

The peak internal pressure $\mathbf{P_i}$ in the barrel occurs coincidentally with the peaking of the base-pressure **P(t)** driving the bullet.

Internal barrel expansion when firing monolithic copper bullets is somewhat more complicated, but with proper bullet design, the internal barrel expansion can be controlled to be ***less*** than would occur in firing a similar lead-cored bullet while the copper bullet still provides adequate barrel obturation.

4.0 Jacketed Lead-Cored Bullets as Performance Benchmark

Conventional hard-jacketed, hard-lead-cored rifle bullets typically obturate conventionally rifled barrels quite well, at least well enough for military and big game hunting rifle applications. We will discuss barrel obturation by our example match-type jacketed rifle bullet made with a softer gilding metal jacket and using a softer core material of essentially pure lead. We will compare the ***radial contact pressure*** $\boldsymbol{\sigma_{rcp}}$ of the bullet exterior gas-sealing surfaces against the inside groove surfaces of the rifle barrel at peak base-pressure **P** when firing our example jacketed SMK bullet versus that of our new copper bullet fired similarly.

From years of experience in precision shooting of match-type jacketed, lead-cored rifle bullets, we know that the default shot-start pressure of **3626 psi** (as used in QL) produces muzzle speed calculations which agree well with our chronograph measurements. For safety, QL recommends using a shot-start pressure of **6500 psi** with any copper bullet. For any given load, this higher shot-start pressure causes much higher calculated peak chamber pressures and produces just slightly higher muzzle speeds. We hope our new bullet design made of half-hard copper requires shot-start pressure no higher than that of the conventional jacketed match bullet.

We will also compare the relative bullet friction characteristics for these two bullets travelling through the rifle barrel as typified by their relative frictions at peak base-pressure **P**. Any significant differences in friction would relate to expected muzzle velocity, barrel wear, barrel fouling, and barrel cleaning differences.

We will use the boat-tailed, 250-grain, 338-caliber Sierra MatchKing bullet as our idea example match-type bullet for comparison of internal ballistics properties. It can be fired quite similarly to our 246-grain monolithic copper bullet for these comparisons. This example lead-cored bullet has a shank length of **0.507 inche**s at its **0.3384-inch** outside diameter. At the rifling-engraved depth of **4.2 thousandths of an inch**, the engraved length of the bullet is **0.603 inches**. The gilding metal jacket measured **0.032-inch** in thickness on a sectioned sample bullet, leaving a lead core diameter of **0.274 inches** in the bullet shank.

5.0 Barrel Obturation by Rifle Bullets

The sealing, or obturation, of the hot gasses produced during combustion of gunpowder propellants is a significant concern in interior ballistics. Leakage of some of the hot gasses past the projectile in the barrel causes minor damage to the barrel and to the projectile. More severe gas blow-by causes more severe damages and contributes to the observed variation in muzzle velocities from one shot to the next. Highest speeds are measured for shots where barrel obturation was best, and reduced speeds are produced when propellant gasses blow by the bullet more at peak base-pressure and thereafter. Early gas blow-by damages the sealing surfaces of the bullet and promotes continued gas leakage during the remainder of that bullet's trip through the rifle barrel.

Direct evidence of damage to a monolithic copper or copper-alloy jacketed bullet is shown by the uniform deposition of metallic copper inside the bore of a rifle barrel toward its muzzle. Elemental copper is vaporized by the hot powder gasses leaking past or blowing-by the bullet at peak base-pressure. This hot copper vapor rapidly expands and cools ahead of the bullet and precipitates out on the cold steel inside surfaces of the barrel toward the muzzle. This thin copper plating is easily removed in routine barrel cleaning, as distinct from "copper fouling" which results from the shearing of copper particles from the bullet due to its encountering rough interior surfaces in the throat or bore of the barrel.

Additional direct evidence of gas leakage can be seen in high-speed video of a rifle being fired. A dense cloud of smoke often obscures the emergence of the bullet itself from the muzzle.

One useful approach to improving barrel obturation during firing is to select barrels made using a rifling pattern designed to improve the sealing of the powder gasses with any type of rifle bullet being fired compared to conventional square-cut rifling. Better obturation is supported by selecting rifling patterns where (**1**) the sides of the rifling lands are sloped significantly outward toward their bases, where (**2**) the bottom inside corners at the edges of the grooves are radiused significantly, and where (**3**) the top edges of the rifling lands are also radiused slightly at both outside corners. Boots Obermeyer's **5R** and Gary Schneider's **P5** rifling patterns come to mind as ideal rifling designs for promoting better bullet obturation. While the author has no direct experience using it, polygonal rifling would also seem to promise better obturation of rifle barrels.

Proper barrel obturation by the rifle bullet is a more complex and difficult problem than most riflemen realize. This is largely because the inside diameter (ID) of the 338-caliber rifle barrel has expanded by **0.0013-inch** at the instant when the peak base-pressure **P** is driving a conventional 338-caliber bullet down the barrel. This is also the very time when gas obturation is most critical and most difficult for the bullet being fired to accomplish. We will show how monolithic copper rifle bullets can be designed to deal with this internal barrel expansion nearly as well as do conventional jacketed, soft-lead-cored match bullets.

5.1 Barrel Obturation with Jacketed Bullets

The soft lead core of a conventional jacketed match bullet readily deforms, or “slugs up,” plastically so that the bullet OD easily matches ***even the pressure-expanded ID of the steel rifle barrel*** at a peak base-pressure **P** of **51 ksi** driving the bullet forward. This accommodating behavior of jacketed, lead-cored rifle bullets explains why riflemen accustomed to shooting jacketed, lead-cored bullets are not generally aware of the significant amount of internal barrel expansion which has been happening all along.

This permanent, plastic, outside diameter enlargement of the jacketed, lead-cored rifle bullet is accompanied by a commensurate plastic foreshortening of the gilding metal jacket and of the lead core within that jacket. As the expanded lead-cored bullet proceeds through the barrel and its base-pressure reduces, the lesser-expanded barrel interior progressively resizes the bullet back to approximately its original dimensions, at some considerable cost in bullet-to-barrel friction.

The initial OD of the jacketed, lead-cored bullet and the unstressed groove ID of the barrel are ***not*** particularly critical dimensions for barrel obturation. In fact, lead-cored bullets of the next larger, or smaller, caliber in the same parent cartridge have sometimes been fired accidentally without damage to the rifle.

With a value of only **2,030,000 psi** for Young’s Modulus of Elasticity (**E_L**) for pure lead, this lead core material has very little “elastic memory” of its pre-stressed shape. This is why lead is called a “dead metal.” You deform it, and it just sits there waiting to be deformed again.

Neglecting the weak, thin gilding metal jacket, the radial contact pressure **σ_{rcp}** of the jacketed bullet against the steel walls of the barrel at peak base-pressure **P** is, as shown earlier,

$$\sigma_{rcp} = \mu_L * S_L + 0.5*[P - S_L]$$

$$\sigma_{rcp} = \underline{25.4\ \text{ksi}}.$$

The total internal pressure **P_i** within the barrel at this point is then

$$\mathbf{P_i = P + \sigma_{rcp} = 51.0 + 25.4 = \underline{76.4\ ksi}}.$$

This radial contact stress **σ_{rcp} = 25.4 ksi** is well above the yield strength (**S_L**) for the pure lead core of just **1.74 ksi**. This pure lead core material is acting almost as an incompressible liquid, at least as far as the axial-to-radial transfer of pressures is concerned.

The peak **σ_{rcp}** value of **25.4 ksi** for a lead-cored rifle bullet gives us a good indication of how much radial contact pressure **σ_{rcp}** might be required for any rifle bullet to seal **51 ksi** of gas pressure very well within any typical rifle barrel using conventional square-cut rifling.

As the base-pressure **P(t)** subsequently drops from its peak value and the rifle barrel begins returning *elastically* to its unstressed ID, the radial contact pressure **σ_{rcp}(t)** at points farther down the barrel becomes

$$\mathbf{\sigma_{rcp}(t) = \mu_L{*}S_L + 0.5{*}[P(t) - S_L] \qquad\qquad [P(t) >> S_L].}$$

As the bullet travels on down the barrel and the base-pressure **P(t)** drops, the less elastically expanded steel toward the muzzle of the barrel re-compresses the lead-cored bullet so that it has almost returned to its nominal bore size on exit from the muzzle. This re-sizing of the lead-cored bullets contributes much to their greater bullet-to-barrel friction compared to copper bullets which do not behave in this way.

If **σ_{rcp}(t)** were always to equal or exceed the base-pressure **P(t)**, one might then say that "perfect mechanical obturation" had been achieved between these two smooth surfaces. These copper-alloy jacketed, lead-cored rifle bullets have about 125 years of development and testing behind them.

6.0 Barrel ID Expansion with Internal Pressure

We look at the internal expansion of our rifle barrels in firing because it is larger than most riflemen would believe and because it is critically important in our analyses of copper bullet performance in interior ballistics.

We assume that the peak chamber pressure in firing our 338 Lapua Magnum rifle cartridges is its CIP-specified Maximum Average Pressure of **60 ksi** (thousands of pounds per square inch). We use the interior ballistics program, QuickLOAD©, to calculate the *peak base-pressure* **P** behind our example bullets at **51 ksi** or **85 percent** of that **60 ksi** peak chamber pressure in this particular example. Either of our example bullets will have travelled only **3.2 inches** down the barrel at the time when the gas pressure acting on the base of that bullet reaches its **51 ksi** peak value. The same peak base-pressure **P** pertains when firing either jacketed, lead-cored bullets or copper bullets of similar weight with similar peak chamber pressures.

We look first at the internal expansion of a stress-free cut-rifled barrel firing our example 338-caliber jacketed, lead-cored bullet, both because this represents a "worst case" and because it is simpler to calculate.

The internal pressure $\mathbf{P_i}$ within the rifle barrel at this point of peak base-pressure **P** must also include the significant peak radial contact pressure $\boldsymbol{\sigma_{rcp}}$ exerted by the lead-cored bullet upon the adjacent walls of internal barrel steel.

As was shown earlier, since **P** greatly exceeds the strength of pure lead $\mathbf{S_L}$ the peak radial contact pressure $\boldsymbol{\sigma_{rcp}}$ for our example jacketed, lead-cored match type bullet is

$$\boldsymbol{\sigma_{rcp} = \mu_L * S_L + 0.5*(P - S_L)}$$

$$\boldsymbol{\sigma_{rcp} = 0.44*1{,}740 + 0.5*(51{,}000 - 1{,}740)}$$

$$\boldsymbol{\sigma_{rcp} = \underline{25.4 \text{ ksi}}.}$$

This radial contact pressure $\boldsymbol{\sigma_{rcp}}$ occurs at this *same time and location* in the barrel where the peak internal gas pressure **P** occurs just behind that bullet. So, the total internal pressure $\mathbf{P_i}$ being contained at this location within the rifle barrel at peak base-pressure **P** on the bullet is the *sum* of this internal gas pressure **P** and the internal mechanical contact pressure $\boldsymbol{\sigma_{rcp}}$ at the base of that bullet:

$$\boldsymbol{P_i = P + \sigma_{rcp} = 51.0 + 25.4 = \underline{76.4 \text{ ksi}}}$$

In normal operation, this peak internal pressure $\mathbf{P_i}$ should not exceed **100 ksi** in good steel rifle barrels for safety reasons.

Our example Heavy Varmint profile 338-caliber match barrels (Bartlein and Schneider) have an outside diameter ($\mathbf{2*r_o}$) of **1.24 inches** at the bullet location corresponding to the peak base-pressure **P**, as very nearly does another **1.25-inch** cylindrical profile Krieger 338-caliber barrel also fitted for our test rifle. We shall use the nominal groove diameter of **0.3380 inches** as the inside diameter (**$2*r_i$**) of each barrel.

Any rifle barrel qualifies mechanically as a thick-walled cylindrical pressure vessel which is not usually stressed in its axial-direction (lengthening or contracting) with increased internal pressure **P_i**. Fortunately, we have Lamé's Equations for calculating analytically the two-dimensional (radial and tangential) stresses and the amount of radial expansion displacement **U(r)** for any element within the steel walls at radius **r ($r_i \le r \le r_o$)** from the axis of the rifle barrel as functions of the total pressure **P_i** being applied internally. These equations hold for any rifle barrels made of isotropic materials (such as any of our barrel-making steels) which have not been pre-stressed and which are not stressed in operation beyond their proportional elastic limits. These initial barrel expansion calculations are made for a stress-free cut-rifled match barrel. The use of a pre-stressed button-rifled barrel blank will be discussed later.

The operative form of Lamé's Equation for finding the radial displacement **U(r)** for an element of the barrel material at any radial location **r** within a pressurized thick-walled cylindrical pressure vessel is:

$$U(r) = (P_i*r/E)*[(1 - \mu) + (1 + \mu)*(r_o/r)^2]/[(r_o/r_i)^2 - 1]$$

where

U(r) = Radial expansion in inches at any radius r from axis ($r_i \le r \le r_o$)
P_i = Internal pressure in psi = 76,400 psi here
E = Young's Modulus of Elasticity = 29,000,000 psi (average for barrel steels)
r_o = Outside radius of cylinder = 0.620 inches here
r_i = Inside radius of cylinder = 0.169 inches here
μ = Poisson's Ratio = 0.30 for barrel steels.

In particular, by setting **$r = r_i$**, the calculated value **$U(r_i)$** becomes the *internal radial* expansion of the inside walls of the pressurized barrel surrounding the base of the jacketed, lead-cored bullet:

U(0.169 inches) = <u>0.000650 inches</u>.

And the internal *diameter* (ID) expansion is **<u>1.300 thousandths of an inch</u>**.

We might also calculate the outside diameter (OD) expansion of this rifle barrel over this same part of the bore by setting $\mathbf{r = r_o}$

U(0.620 inches) = 0.000262 inches.

As expected, the outside diametral (OD) expansion over this point in the barrel is *much less* at just **0.525 thousandths of an inch**, or **40-percent** of the ID expansion.

However, this does show how external strain gauges can allow indirect laboratory measurement of base-pressures behind the bullet up and down a cylindrical test barrel.

Had our 338-caliber barrel been of much lighter profile with a **0.75-inch** OD at this point ahead of the chamber swell, for example, the *internal diameter* expansion at this point of peak base-pressure would have been even greater at **1.611 thousandths of an inch**.

Note that these ID and OD elastic expansions with internal pressure $\mathbf{P_i}$ are not dependent on the quality, heat treatment, surface hardness, or strength ratings of the barrel steel. These elastic internal barrel expansions are quite accurately calculated provided the barrel material has no *previously implanted stresses* caused by its having been stressed earlier beyond its elastic limit as in button rifling or in proof testing.

As we will show later in **Section 7.1** on calculating the radial contact pressure for our monolithic copper bullet, the barrel's peak internal radial expansion $\mathbf{U(r_i)}$ when firing that new copper bullet similarly through a stress-free, cut-rifled 338-caliber barrel is less at

$\mathbf{U(r_i)}$ **=** <u>**0.000578 inches**</u>

for an internal diameter expansion of <u>**1.156 thousandths of an inch**</u>.

Button-rifling is a precision barrel rifling process which *necessarily* over-stresses a thin-walled cylinder of the internal barrel steel surrounding the bore and leaves behind a residue of implanted tangential (hoop) stresses within the steel surrounding that internal cylinder which actually *does* reduce subsequent bore expansions with internal pressures. Without conducting a detailed analysis, we estimate that button-rifled barrels will typically expand internally by about **2/3** as much as would a similar profile and caliber stress-free cut-rifled barrel at similar internal pressures. That is, the implanted hoop stresses will *cancel* about the first **20** to **25 ksi** of any and all internal pressures subsequently applied. For this very reason, a similar over-stressing operation, called “autofrettage,” is routinely performed either hydraulically or mechanically during the manufacturing of artillery tubes.

We should also note that the internal-pressure expansion of the rifle barrel has the same basic timing as the chamber pressure curve. The peaking impulse of this chamber pressure itself looks like the first half of a sinusoidal pressure wave of about **1200 hertz**. This becomes the fundamental driving frequency for many types of barrel vibrations in firing.

This rapid internal expansion of the barrel at peak base-pressure generates a ***shear pressure wave*** **(of bore expansion)** which travels up and down the length of the rifle barrel, propagating axially at between **10,230 fps** (for stainless steel) to **10,630 fps** (for AISI 4340 steel). Transverse barrel vibration waves are also shear waves and travel up and down the barrel at these same speeds, but longitudinal acoustic pressure waves travel about 50-percent faster as plane wave-fronts within a thin steel rod such as any of our rifle barrels.

In these *shear pressure waves*, the particle motion is perpendicular to the axial direction of wave propagation. These shear waves reflect back and forth from the impedance changes at the muzzle and breach ends of the rifle barrel and sum algebraically at any point along the barrel. Best rifle accuracy is obtained when the multiply-reflected shear pressure waves cause ***minimal internal barrel expansion*** at the muzzle just at the time of bullet exit. This appears to be the physical basis of empirical ***Optimum Barrel Time*** calculations.

7.0 Barrel Obturation with Copper Bullets

Monolithic copper bullets can be designed so that they achieve ***effective obturation*** of the hot powder gasses throughout the interior ballistics portion of their firing process. A 338-caliber copper ULD bullet of my own patented design is used here as an illustrative example.

Some monolithic bullet designs utilize a sequence of narrow, over-diameter gas sealing rings which are designed always to be plastically compressed during rifling engravement. A similar approach is used in artillery shell designs because of their hard steel shell casings. Plastic deformation in compression always includes a *maximum elastic compression* at the elastic limit for the material of the sealing rings.

A downside of using multiple small gas sealing rings for monolithic copper rifle bullets is the necessarily higher aeroballistic drag induced by the multitude of secondary shock waves which these sealing rings invariably throw off during supersonic and transonic flight.

Another disadvantage of using narrow sealing rings for barrel obturation is that their elastic “working length” in providing contact pressure against the inside of the barrel is only the compressed height of the sealing rings themselves, typically just a few millimeters on a large artillery shell, as opposed to the full radius of an engraved shank of a monolithic bullet or of a much wider driving/sealing band.

These are among the reasons we selected a wider rear driving/sealing band design for our monolithic copper ULD bullets.

Let us start by looking at the rifling-engraved bullet at the moment of peak chamber pressure (**60.0 ksi**, in this example) which occurs when the bullet has travelled just **3.2 inches** down the rifle barrel from its initial position. According to the interior ballistics program *QuickLOAD©* the peak base-pressure **P** accelerating our example 246-grain copper 338-caliber bullet is

$$\mathbf{P = 51.0\ ksi.}$$

For the first **S = 40 ksi** of the peak base-pressure **P** on our copper bullet, the expansion of the bullet is *elastic*, but since **P > S**, the remaining excess pressure (**P – S**) produces *plastic* deformation:

$$\boldsymbol{\sigma_{rbp} = \mu * S + 0.5*(P - S) = \underline{18.7\ ksi}}$$

with Poisson’s Ratio **μ** equal **0.33** for this copper material and the *plastic* flow treated as a liquid flow for pressure transfer purposes.

Now, let us consider what happens when this unstressed copper aperture-sealing disc *exactly fits* the interior of the rifle barrel at this point where maximum base-pressure **P** is applied to it. That is, let us assume for the moment that it has **zero** unloaded (static) radial contact pressure $\boldsymbol{\sigma_{r0}}$. Let us also assume for the moment here that the much stronger steel walls of the rifle barrel do not move outward with these interior pressure stresses.

When the base-pressure **P** is applied, the radial contact pressure $\boldsymbol{\sigma_{rcp}}$ is

$$\boldsymbol{\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} = 0 + 18.7 = 18.7\ ksi}$$

Perhaps we could achieve more perfect barrel obturation by instead starting with a positive static contact pressure $\boldsymbol{\sigma_{r0}}$. We could have radially compressed an over-diameter copper bullet by an amount **Δr** as it entered the throat of the barrel so that its static contact pressure $\boldsymbol{\sigma_{r0}}$ is

$$\boldsymbol{\sigma_{r0} = E^*(\Delta r)/r \leq S}$$

where the upper limit **S** (**40,000 psi** here) is the rated yield strength of our half-hard copper bullet material, and **r** is the radius of our unstressed copper bullet's gas-sealing surface.

One should design the maximum outside diameter (OD) of the copper bullet's shank or of its driving/sealing band so that, within bullet manufacturing tolerances, the bullet OD will always be at least as large as the maximum groove inside diameter (ID) for standard specification rifle barrels of each caliber.

With a production tolerance of **+/- 0.0001 inch** in bullet diameter for these CNC-turned half-hard copper bullets, we specify a rear driving/sealing band OD **0.0006-inch** larger than the **nominal groove ID** for standard barrels of this 338 caliber. Then, when specifying the chambering reamer design, we specify a ball seat ("freebore") inside diameter **0.0008-inch** larger than this nominal groove ID to minimize gas blowby before the bullet enters the conical throat of the rifle barrel.

Groove ID tolerance in match grade barrel production can be specified to fall between the **nominal groove ID** for that caliber and the **nominal groove ID+0.0002 inch**, with less than **0.0001-inch** variation, end to end, and only tightening toward the muzzle. Thus, any production bullet within specifications will freely enter the ball seat and ***statically*** seal the throat of the fitted production barrel blank during firing.

For our 338-caliber copper bullet, **Δr** is nominally **0.0003 inch**, and **r** is **0.1693 inch**, so the copper rear driving band, after passing through the throat of the barrel, has a compressive radial elastic strain ratio $\boldsymbol{\varepsilon_{r0}}$ of

$$\boldsymbol{\varepsilon_{r0} = (\Delta r)/r = (0.0003/0.1693) = 0.001772 < \varepsilon_{Max}}$$

The “half hard” copper from which these bullets are manufactured has a guaranteed minimum yield strength rating **S** of **40,000 psi** and a Modulus of Elasticity **E** of **16,900,000 psi**. Thus, its maximum possible elastic strain ratio $\boldsymbol{\varepsilon_{Max}}$ is given by

$$\boldsymbol{\varepsilon_{Max} = S/E = 40{,}000/16{,}900{,}000 = 0.002367}$$

The selected nominal radial compression **Δr = 0.0003 inches** produces **75 percent** of the *maximum possible* radial stress pre-load, and

$$\boldsymbol{\sigma_{r0} = E^*(\Delta r)/r = \underline{30.0\ ksi}.}$$

If the production and wear tolerances stack so that the radial compression **Δr** has its minimum value of **0.00025 inches**,

$$\boldsymbol{\sigma_{r0} = 25.0\ ksi}.$$

If the production and wear tolerances stack so that **Δr** has its maximum value of **0.00035 inches**,

$$\boldsymbol{\sigma_{r0} = 35.0\ ksi}$$

The total radial contact pressure now looks pretty good at a nominal

$$\boldsymbol{\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} = 30{,}000 + 18{,}700 = 48.7\ ksi}$$

which is almost the base-pressure **P = 51.0 ksi**, and would produce “perfect mechanical obturation” as mentioned earlier.

However, we have to go back and account for the steel walls of the barrel expanding by **Δr = -0.000434 inches** due *solely* to peak base-pressure **P**. The change $\boldsymbol{\sigma_{rexp}}$ in copper radial bearing stress due to internal expansion of the steel barrel would be given by

$$\boldsymbol{\sigma_{rexp} = E^*(\Delta r)/r = 16{,}900{,}000^*(-.000434/0.1690) = -43.4\ ksi}$$

And the total radial bearing stress $\boldsymbol{\sigma_{rcp}}$ of the copper bullet inside the expanded barrel is now given by

$$\boldsymbol{\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} = 48{,}700-43{,}400 = 5.3\ ksi}$$

Now we begin to see why copper bullets have difficulty sealing the powder gasses effectively. In fact, when we iterate this calculation with barrel internal expansion for

$$\boldsymbol{P_i = 51.0 + 5.3 = 56.3\ ksi,\ etc.}$$

the radial contact pressure **σ_{rcp}** of this copper bullet converges on a small *negative* value, **-300 psi**, meaning *no contact*, even when including the small **1.7 ksi** contact pressure (as formulated later) due to centrifugal force at this same time of peak base-pressure **P**.

Fortunately, there is another way to increase this radial bearing pressure for monolithic copper bullets by a designed amount, and that is simply by drilling their bases axially to port, or duct, the base-pressure inside the gas sealing portion of those bullets.

7.1 Base-Pressure Ducting by Base-Drilling of Copper Bullets

Large potential (unconstrained) bullet diameter increases **$U(r_o)$** or corresponding (constrained) increases in its radial contact pressures **σ_{rcp}** are available by porting, or ducting, the peak base-pressure **P** forward to within the body of the monolithic copper bullet by axially base-drilling those bullets. To be effective in improving bullet obturation of the barrel, this base-drilling must pass completely through the boat-tail and at least part way through under the rear driving/sealing band of this bullet design. The idea of ducting the base-pressure into a hollowed-out base of a rifle bullet goes back in history at least to 1849 and the under-sized, hollow-base, lead musket ball developed by French Army Captain Claude-Étienne **Minié** for the muzzle-loading rifled muskets of that era.

The photograph below illustrates the difference in gas sealing efficiency for prototype monolithic copper 338-caliber ULD bullets fired at 3000 fps into a swimming pool and recovered for study.

These prototype bullets have **0.3302-inch** diameter bore-riding shanks and **0.3382-inch** diameter rear driving/sealing bands. The angle of incidence into the water was about 45 degrees, which explains the neatly curled copper ogives. The test barrel was a 10-inch twist Krieger barrel with a conventional six-narrow-land cut-rifling pattern. The bore diameter of the barrel was **0.3302 inches**, and the groove diameter was **0.3380 inches**.

The top bullet is an un-fired example. The second bullet was fired into the pool, but it had not been base-drilled. The bottom two fired bullets had been base-drilled, using a **0.166-inch** diameter drill, to the depths indicated by the black annotation marks.

Note the very poor gas sealing of the second (un-drilled) bullet. The engraved groove-diameter copper bullet contacts only the middle third of the rifling grooves in the barrel. Compare that to the two base-drilled bullets also shown. Their engraved rear driving/sealing bands look similar to the shanks of recovered soft-jacketed, soft-lead-cored bullets. Also, notice the evidence of elastic bullet expansion just to the shoulder depth of the base-drilling indicated by the annotation marks on the bottom two bullets.

Porting the base-pressure inside the bullets caused none of the plastic deformations seen in these fired bullets other than the slightly more noticeable rifling scratch marks seen on the bore-riding shanks of the deep-drilled bullets. We have never seen any evidence of unburned powder packing into these hollow bases in the many recovered test bullets which had been base-drilled with different drill diameters and to different drill depths and were fired using various different rifle propellants.

With proper base-drilling, the rearmost aperture-sealing portion of the rear driving/sealing band becomes a thick-walled cylindrical pressure vessel for which Lamé's Equations can again be used to calculate internal stresses and radial displacement as functions of the base-pressure **P** ducted internally.

The operative form of Lamé's Equation for radial displacement **U(r)** is:

$$U(r) = (P*r/E)*[(1 - \mu) + (1 + \mu)*(r_o/r)^2]/[(r_o/r_i)^2 - 1]$$

where

U(r) = Radial expansion at radius r from axis of cylinder ($r_i \le r \le r_o$)
P = Internal pressure in psi = 51,000 psi here
E = Young's Modulus of Elasticity = 16,900,000 psi (Cu)
r_o = Outside radius of cylinder = 0.1693 inches here
r_i = Inside radius of cylinder = 0.0830 inches here
μ = Poisson's Ratio = 0.33 for copper (Cu).

Here, we are calculating the maximum temporary elastic radial expansion **U(r)** of the rear driving band of our 338-caliber copper ULD bullet as a function of the radial position **r** from the cylinder axis for $r_i \le r \le r_o$.

In particular, we want to find the potential unconstrained radial expansion at the *outside surface* of the copper bullet with this internal gas pressure. Setting $r = r_o$ for this special case, Lamé's Equation above reduces to:

$$U(r_o) = (2*P*r_o)/\{E*[(r_o/r_i)^2 - 1]\}$$

Using our numerical values for this copper 338-caliber ULD bullet, we have:

$$U(0.1693) = (2*51{,}000*0.1693)/(16{,}900{,}000*3.1606)$$
$$U(0.1693) = \underline{0.000323\text{ inches}}.$$

Thus, the outside *diameter* of the rear driving band could temporarily increases by a calculated **0.000647 inches** when a hydrostatic gas pressure of **51.0 ksi** is applied to the inside of the obturating surface of the **0.166-**inch ID hollow-based copper bullet. This radial expansion is *purely elastic* because **$U(r_o)$ = 0.000323 inches** is less than the maximum elastic radial expansion of

$$\Delta r_{Max} = 0.1693*S/E = 0.000401\text{ inches}$$

for these half-hard copper bullets. When the internal pressure drops to **zero psi** (actually less than ambient air pressure) during subsequent aeroballistic flight, the bullet returns to its (engraved) original shape.

This temporary diameter increase significantly improves the obturation of the monolithic copper ULD bullet ***exactly when it is most needed***. Fired test bullets recovered from the waters of a swimming pool show ***essentially perfect obturation*** of these base-drilled bullets forward to the shoulder depth of the internal drilling.

Except for the penalty in ballistic coefficient (BC) caused by the reduction in bullet weight, no other ill effects in bullet behavior are caused by base-drilling. Base-drilling simultaneously improves the gyroscopic stability of the monolithic copper bullets and allows the lighter-weight bullets to be fired at higher muzzle speeds.

Now, if we constrain this potential bullet expansion $\mathbf{U(r_o)}$ due to base-drilling to exert instead a radial pressure $\boldsymbol{\sigma_{rbd}}$ against the inside of the barrel walls, with **Δr = 0.000323 inches**, the elastic expansion when using this **0.166-inch** base-drill diameter, this radial contact pressure would be

$$\boldsymbol{\sigma_{rbd} = E^*(\Delta r)/r = 16{,}900{,}000^*(.000323/0.1693) = 32.3\ \text{ksi}}.$$

If we reduce the base-drill diameter ($\mathbf{2^*r_i}$) from **0.166-inch** to **0.125-inch**, the expression $\mathbf{[(r_o/r_i)^2 - 1]}$ in the denominator of Lamé's Equation increases from **3.1606** to **6.3376**, just about cutting the potential elastic diametral expansion in half, from **0.647 thousandths** to **0.3225 thousandths of an inch.**

Correspondingly, with **Δr = 0.0001612 inches** with the smaller base-drill,

$$\boldsymbol{\sigma_{rbd} = E^*(\Delta r)/r = \underline{16.1\ \text{ksi}}}.$$

The timing of this base-pressure-ducting bullet expansion is the same as that of the inertial-force-driven bullet expansion, a reduced amplitude and slightly delayed version of the chamber pressure curve.

With the **0.166-inch** drill size, the total radial contact pressure $\boldsymbol{\sigma_{rcp}}$ of the copper bullet would be

$$\boldsymbol{\sigma_{rcp} = 5{,}300 + 32{,}300 = 37.6\ \text{ksi}},$$

and with only a **0.125-inch** drill size, it is

$$\sigma_{rcp} = 5{,}300 + 16{,}100 = \underline{21.4\ ksi}.$$

Thus, the **0.125-inch** base drill size looks adequate for this 338-caliber copper bullet, and perhaps for similar sizes of copper ULD bullet designs. The **4-percent** bullet weight penalty of **10.2 grains** for this smaller base-drilling is an acceptable trade with this **256-grain** (solid-weight) 338-caliber monolithic copper ULD bullet design for the resulting improvement in gas sealing. However, we must scale down this base-drill diameter for smaller-caliber copper bullets of this same ULD design to minimize weight reduction.

Now, if we recalculate the internal expansion **$U(r_i)$** of the steel barrel at peak base pressure **P** for this copper bullet with its reduced internal bearing pressure (compared to the lead-cored bullet), we find

$$P_i = 51.0 + 21.4 = 72.4\ ksi$$

$$U(r_i) = -\Delta r = 0.000616\ inches.$$

While this amount of barrel internal expansion would appear to exceed the elastic limit of expansion of the 338-caliber copper bullet (**0.0004 inches**), it does *not* do so because the bullet had been elastically pre-compressed radially by **0.0003 inches**.

Then $\sigma_{rexp} = E^*(\Delta r)/r = 16{,}900{,}000^*(-.000616/0.1690) = -61.6\ ksi$

So, the summed radial contact pressure **σ_{rcp}** for the copper bullet (including the small contribution due to centrifugal force calculated later) is now

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} + \sigma_{rbd} + \sigma_{rcf}$$

$$\sigma_{rcp} = 30.0 + 16.8 – 61.6 + 16.1 + 1.7 = 3.0\ ksi.$$

Iterating this calculation for barrel internal expansion with copper bullet bearing pressure converges on

$$\sigma_{rcp} = 30.0 + 16.8 - 57.8 + 16.1 + 1.7 = \underline{6.8\ ksi}.$$

We believe this **<u>6.8 ksi</u>** radial contact pressure will produce effective barrel obturation.

So, an internal radial barrel expansion with these copper bullets of

$$U(r_i) = \Delta r = \underline{-0.000578\ inches}$$

produces a radial bearing pressure **σ_{rcp}** of **6.8 ksi**, which is **27-percent** of the **25.4 ksi** produced when firing the jacketed, lead-cored match bullet.

Thus, the internal diameter of the barrel expands by just **1.156 thousandths of an inch** at peak base-pressure when firing our base-drilled copper bullet, as compared with **1.300 thousandths** when firing the conventional lead-cored bullets similarly.

Increasing the diameter of the base-drill used in making these copper bullets would increase this radial contact pressure **σ_{rcp}**, but not by as much as one might hope, due to the additional internal barrel expansion which would then occur during firing.

If we instead used the original base-drill diameter of **0.166 inches**,

$$\sigma_{rcp} = 30.0 + 16.8 - 65.9 + 32.3 + 1.7 = 14.9 \text{ ksi}$$

While this radial contact pressure looks better at **59 percent** of that with the conventional lead-cored bullet, the internal barrel expansion of **1.318 thousandths of an inch**, due to this peak internal pressure **P_i** of **83.3 ksi** (encroaching upon our **100 ksi** safety limit for unstressed cut-rifled barrels), would be greater than the barrel expansion of **1.300 thousandths** with the conventional bullet, and that copper bullet would weigh only **240 grains**. These are the reasons for using only the **0.125-inch** base-drill size.

7.2 Summary of Barrel Obturation

Analyzed in terms of bullet expansion into the pressure-expanded rifling grooves of the barrel, the lead-cored bullet has exceeded its elastic limit of **1.7 ksi** by **23.7 ksi** at peak base pressure **P**, so it expands *plastically* to fill the **0.000650-inch** radially expanded bore of the rifle barrel.

The elastic limit of radial expansion for the half-hard copper 338-caliber bullet is **0.000400-inch** in addition to the re-expansion of the **0.000300-inch** initial radial compression of its oversize driving band. As their sum exceeds the radial internal expansion of the steel barrel **0.000578 inches**, this copper bullet does not distort *plastically* other than its rifling engravement.

Each bullet will distort either *plastically* or *elastically* so as to fill and seal the radially expanded rifling grooves with almost equal effectiveness.

While study of the elasticity and strength of materials is far more complex than this elementary discussion might indicate, this analysis is sufficient for our purposes here.

8.0 Barrel Friction with Copper and Jacketed, Lead-Cored Bullets

We will compare the friction of the two example bullets at the time of peak base-pressure, which is also when peak barrel internal expansion occurs. This comparison will also indicate any differences in friction characteristics to be expected elsewhere in the barrel.

From the values of radial contact pressure $\boldsymbol{\sigma_{rcp}}$ calculated earlier, we will formulate the total normal contact force $\mathbf{F_C}$ distributed over the contact area $\mathbf{A_C}$ for each bullet. We can ignore the rifling lands for now and make "smooth bore" friction calculations for the nominal groove ID of $\mathbf{D_g}$. We then find the nominal force-of-friction $\mathbf{F_f}$ resisting bullet motion by multiplying this normal force $\mathbf{F}$ times the coefficient of sliding friction $\mathbf{C_f}$ for the outer material of each bullet sliding smoothly over clean, dry barrel steel.

The actual bullet to barrel coefficient of dynamic friction $\mathbf{C_f}$ experienced during any given shot is far too complicated to ever be dealt with analytically. The interior barrel surfaces are seldom the "clean, dry steel" mentioned above, especially after the first shot, and even the type of barrel steel selected and its micro-finish might be important variables here. Friction modifiers are also used at times.

Yet, somehow rifles can often fire bullets which exit the barrel uniformity enough to produce repeatable precision at long ranges. Our purpose here is only to develop some basic physical relationships to allow comparative friction evaluations of our two different bullets, not an attempt to provide hard numerical performance predictions.

For comparison purposes, we will calculate a friction-equivalent partial base-pressure $\mathbf{P_f}$ for each type of bullet.

From the bore obturation studies of these two bullets, we found their radial contact pressures $\boldsymbol{\sigma_{rcp}}$ at **51.0 ksi** peak base-pressure to be:

$$\boldsymbol{\sigma_{rcp} = 22.4\ \text{ksi}} \qquad \textbf{(Jacketed bullet)}$$

$$\boldsymbol{\sigma_{rcp} = 6.8\ \text{ksi}} \qquad \textbf{(Copper bullet)}$$

The contact area $\mathbf{A_C}$ of each bullet can be formulated for a smooth bore as:

$$A_C = \pi * D_g * L_C$$

where D_g = **Nominal Groove Diameter = 0.3380 inches**

L_C = **Axial Length of Contact Patch**

$L_C = 0.507$ **inches (Jacketed bullet)**

$L_C = 0.233$ **inches (Copper bullet).**

Then $A_C = 0.5384$ **sq. in. (Jacketed bullet)**

$A_C = 0.2474$ **sq. in. (Copper bullet)**

The total normal contact force F_C distributed over the contact area A_C is then

$$F_C = \sigma_{rcp} * A_C$$

and the aggregate axial force-of-friction F_f opposing bullet motion is then

$$F_f = C_f * F_C = C_f * \sigma_{rcp} * A_C.$$

Now, the partial base-pressure P_f equivalent to this axial force-of-friction F_f spread over the cross-sectional area of the bore A_B is

$$A_B = (\pi/4) * D_g^2 = 0.08973 \text{ sq. in.}$$

$$P_f = F_f / A_B = C_f * \sigma_{rcp} * (A_C / A_B)$$

$$P_f = C_f * (4 * L_C / D_g) * \sigma_{rcp}.$$

Using the values for our two bullets:

$$P_f = 0.22 * (4 * 0.507 / 0.338) * 22.4 \text{ ksi}$$

$$= 29.6 \text{ ksi} \qquad \text{(Jacketed bullet)}$$

and

$$P_f = 0.36 * (4 * 0.233 / 0.338) * 6.8 \text{ ksi}$$

$$= 6.8 \text{ ksi} \qquad \text{(Copper bullet)}$$

While these two “friction” pressures are neither accurate nor realistic because we are ignoring so many other effects, we wish only to show the importance of *minimizing the bullet-to-bore contact area* A_C in controlling bullet friction. The higher coefficient of friction C_f of the copper bullet can be more than compensated by its having a much shorter contact length L_C.

At muzzle speeds of **2960 fps**, either bullet carries about **4850 ft-lbs** of kinetic energy **KE** at the muzzle. The average base-pressure $\mathbf{P_{ave}}$ (averaged over bullet position in the barrel) can be found by dividing the muzzle energy **KE** by the *Volume of the Bore* $\mathbf{V_B}$ swept by the base of the bullet while being propelled by this spatially averaged base-pressure throughout its bullet-travel distance here of **25.9 inches**.

$$\mathbf{V_B = (\pi/4)*0.338^2 *25.9\ in = 2.3239\ cu.\ in.}$$

$$\mathbf{P_{ave} = KE/V_B = 12*4850/2.3239 = 25.04\ ksi}$$

This spatial-average base-pressure $\mathbf{P_{ave}}$ seems reasonable at **49.25 percent** of the accurately estimated peak base-pressure **51.0 ksi**, and establishes the pressure “peaking ratio” of **1/0.4925 = 2.0305** for these bullets in this firing condition.

From a long ago Army technical study reported in Hatcher’s Notebook measuring the total energy disposition in firing a 30-’06 military cartridge, we expect the energy lost to bullet/barrel friction to be about **7.4 percent** of the total energy released from the propellant with similar jacketed, lead-cored bullets.

QL calculates the energy efficiency for our 338 load at **29.2 percent**, so the total energy released by the powder is **16,610 foot-pounds** for each shot. Thus, the spatially averaged partial base-pressure due to friction $\mathbf{P_{f(ave)}}$ for the jacketed bullet should be approximately

$$\mathbf{P_{f(ave)} = 0.074*12*(16,610/2.3239) = 6,348\ psi}.$$

and the *peak* friction-equivalent partial pressure $\mathbf{P_f}$ for jacketed bullets should have been about

$$\mathbf{P_f = 6348/0.4925 = 12,887\ psi} \qquad \textbf{(Jacketed bullet).}$$

Scaled by our comparative *friction pressure* calculations above, we might then expect that for our copper bullet this *peak* friction pressure might be about

$$\mathbf{P_f = (6.8/29.6)*12,887 = 2,939\ psi} \qquad \textbf{(Copper bullet).}$$

This would also indicate a kinetic energy loss due to friction of just **1.7 percent** with our copper bullets.

This simple comparison indicates that we need not be overly concerned about decreased barrel life or increased barrel cleaning problems due to friction when switching from shooting jacketed, lead-cored match bullets to shooting properly designed and manufactured half-hard copper bullets.

9.0 Shot-Start Pressure with Copper Bullets

Many riflemen are justifiably concerned about perhaps encountering higher shot-start pressures when switching from firing conventional jacketed, lead-cored rifle bullets to using stout monolithic copper bullets, or when using rifles having differing chamber throat angles, rifling patterns, and rifling twist-rates. All else being equal, higher shot-start pressures ***do*** cause significantly higher chamber pressures while producing only marginal gains in muzzle speed.

9.1 Engravement of the Rifling Lands into the Bullet

The amount of base pressure $\mathbf{P_E}$ necessary to engrave the rifling into the bullet depends upon (**1**) the yield stress **S** of the bullet material, (**2**) the throat half-angle **Φ** of the barrel, and (**3**) the fraction **k** of the width of each rifling land divided by the sum of the land-width plus the groove-width for the rifling pattern of the barrel being used. The depth of the rifling grooves is always great enough to produce permanent engraving marks as *plastic* deformations on the bullet shank or driving band. Thus, the radial stress $\boldsymbol{\sigma_r}$ needed to impress each land into the bullet depends only upon the yield stress **S** of the material being engraved. A radial stress $\boldsymbol{\sigma_r}$, equal to or exceeding **S**, applied all around the perimeter of the bullet would compress that entire perimeter first *elastically* and then *plastically* (and would temporarily or permanently lengthen that compressed bullet correspondingly).

Let us measure the land and groove widths at *one-half the groove depth* for our example 10-inch twist Krieger rifle barrel. According to geometry, the width at half-depth of a single land and groove pair for our rifle barrel would then be **(π/N)*0.3340 inches**, where **N = 6** (here) is the integer number of these land/groove pairs in the selected rifling pattern. So, each land/groove pair is **0.175 inches** in width in this example rifle barrel.

For a match-type rifle barrel with really narrow lands, the land-width fraction **k** might be as small as about **0.25**, and the fraction **k** might approach **0.5** for a typical military or factory-type hunting rifle. As our rifling lands seem to be about **52 thousandths** wide, we estimate **k = 0.30** for our example barrel and will use that value for our calculations here.

The rifling lands act to localize, or concentrate, the radial stress σ_r by a factor of **1/k** so that a full-perimeter radial compression stress of only

$$\sigma_r = k*S = 0.3*40{,}000 = 12.0 \text{ ksi}$$

is required to engrave these rifling lands fully into the copper bullet.

The base-pressure P_E required to engrave the rifling lands produces an axial stress σ_a equal to that base-pressure

$$\sigma_a = P_E$$

But this axial stress σ_a produces a radial stress σ_r according to

$$\sigma_r = \mu*\sigma_a$$

where **μ = 0.33** is Poisson's Ratio of lateral shrinkage for this copper bullet material under tensile loading.

So that the required engraving base-pressure P_E is given by

$$P_E = \sigma_a = \sigma_r/\mu = k*S/\mu$$

Or, using our example values for this copper bullet in this rifle barrel,

$$P_E = 0.30*40{,}000/0.33 = 36{,}364 \text{ psi.}$$

This looks pretty fierce, but we have a mechanical advantage here which we have not yet considered.

The front slopes of the ends of the rifling lands slope back at a shallow throat half-angle **Φ** as measured from the axis of the bore. As a simple tool, this "wedge angle" provides a mechanical advantage given by the reciprocal of the trigonometric tangent of the throat angle **Φ**. Our custom 338 Lapua Magnum chambering reamer cuts a fairly steep throat half-angle **Φ** of **4 degrees**, which has a co-tangent of **14.3**, and which is still quite a significant mechanical advantage. This mechanical advantage would be **2.67** times greater at **38.2** for a more typical **1.5 degree** throat angle.

So, neglecting friction, the basic shot-start pressure P_E required to engrave the rifling for our example rifle and our copper bullet is now

$$P_E = \text{Tan}(\Phi)*k*S/\mu = 0.06993*36{,}364 = 2{,}543 \text{ psi}$$

This basic shot-start pressure $\mathbf{P_E}$ is enough to engrave any reasonable rifling pattern into a monolithic copper bullet to the full depth of the grooves, but without yet allowing either for barrel internal expansion or friction effects.

9.2 Compression of the Rear Driving Band to Groove ID

The rear driving bands of our example copper ULD bullets are made **0.0006-inch** larger in OD than the **0.3380-inch** nominal ID of the grooves in a new (unworn) standard 338-caliber rifle barrel. This extra copper material is elastically compressed within the rifling grooves during the plastic engravement of the nearby lands into that driving band.

The radial compressive strain ratio $\boldsymbol{\varepsilon_r}$ required to compress this extra copper material within the grooves is given by

$$\boldsymbol{\varepsilon_r = 0.0006/0.3380 = 0.001775}$$

The radial stress $\boldsymbol{\sigma_r}$ required to produce this elastic strain ratio $\boldsymbol{\varepsilon_r}$ in our example copper bullet is

$$\boldsymbol{\sigma_r = E*\varepsilon_r = 16{,}900{,}000*0.001775 = 30.0\ \text{ksi}}$$

Now, we can combine **(1 – k)** times this elastic radial stress in the grooves of the rifling pattern with **k** times the radial stress **S** shown above for plastically engraving the lands to find the ***total radial stress*** $\boldsymbol{\sigma_r}$ exerted on the copper bullet by the steel throat of the barrel to be

$$\boldsymbol{\sigma_r(\text{Total}) = (1 - k)*30.0 + k*S}$$

$$\boldsymbol{\sigma_r(\text{Total}) = 0.70*30.0 + 0.30*40.0 = 33.0\ \text{ksi}}$$

Scaling the barrel radial expansion at peak base pressure **P** calculated earlier by using Lamé’s Equation, shows that our steel rifle barrel will expand internally here in its throat during rifling engravement by

$$\boldsymbol{U(r_i) = (33.0/76.4)*0.000650\ \text{inches}}$$

$$\boldsymbol{U(r_i) = 0.000281\ \text{inches.}}$$

Here we are ignoring the tiny amount of barrel expansion due to the shot-start gas pressure itself.

The radial expansion of the barrel is linearly proportional to the pressure applied to it internally, whether that pressure is supplied by internal gas pressure or by direct surface-to-surface contact pressure $\boldsymbol{\sigma_r}$ with the base of our the bullet. Our example barrel is nearly cylindrical in outer profile here and has the same OD over its throat (**1.24 inches**) as it has **3.2-inches** forward over the point of peak base-pressure application, so that the non-linear portions of Lamé's Equations remain unchanged.

The radial compressive stress relief in the copper bullet caused by this amount $\mathbf{U(r_i)}$ of internal radial barrel expansion is then given by

$$\boldsymbol{\varepsilon_r = U(r_i)/r = -0.000281/0.1690 = -0.0016627}$$

$$\boldsymbol{\sigma_r = E^*\varepsilon_r = -0.0016627^*16{,}900{,}000 \text{ psi}}$$

$$\boldsymbol{\sigma_r = -28.1 \text{ ksi.}}$$

So, the net total radial stress exerted by the copper bullet upon the inside of the steel barrel throat now becomes

$$\boldsymbol{\sigma_r(\text{Total}) = 0.70^*30.0 + 0.30^*40.0 - 28.1 = 4.9 \text{ ksi}}$$

The total axial stress $\boldsymbol{\sigma_a}$ needed to produce this total radial stress $\boldsymbol{\sigma_r}$ is then

$$\boldsymbol{\sigma_a = \sigma_r/\mu = 4{,}900/0.33 = 14.85 \text{ ksi.}}$$

The throat angle of **4 degrees** produces the same mechanical advantage of **Cotangent(4 degrees) = 14.30**, both for the plastic engraving of the lands and for the elastic compressing of the copper bullet material within the grooves, so that the net combined shot-starting pressure required both for compressing and engraving the copper bullet in the throat $\mathbf{P_{CE}}$, neglecting friction, becomes

$$\boldsymbol{P_{CE} = \text{Tan}(\Phi)^*\sigma_a = 0.06993^*14.85 \text{ ksi} = 1{,}038 \text{ psi.}}$$

Now we need to find how much the friction between the outside of the bullet and the inside of the rifle barrel increases this basic combined shot-start engraving and compressing pressure.

9.3 Friction of Copper Bullet in Throat of Steel Barrel

We showed earlier that a peak base-pressure **P = 51 ksi** behind our example 338-caliber, base-drilled copper ULD bullet produced a peak radial contact pressure $\boldsymbol{\sigma_r = 6.8 \text{ ksi}}$, which is significantly less than the peak radial contact pressure of **25.4 ksi** which would be produced when firing similarly an ideal soft-jacketed match bullet made with a soft core of pure lead.

This earlier calculation even allowed for the internal expansion of the steel rifle barrel caused by that peak base-pressure **P** of **51.0 ksi**, with the added contact pressure $\boldsymbol{\sigma_r}$ of **6.8 ksi**.

Thus, it would be reasonable to estimate that

$$\boldsymbol{\sigma_r(t) = (6.8/51.0)*P(t) = 0.1333*P(t)}$$

for lower, off-peak base-pressures **P(t)** as well.

These numbers are basically "smooth-bore" values which do not consider the spin-up of the accelerating bullet which requires only a negligible base-pressure. We might term this ratio of pressures $\boldsymbol{R_P = 0.1333}$ and think of it as a sort of "aggregated" version of Poisson's Ratio **μ** for this copper bullet.

The total contact area $\boldsymbol{A_C}$ between the outside of the dual-diameter copper bullet and the inside of the barrel steel can be formulated as

$$\boldsymbol{A_C = \pi*[(1 - k)*D_g*L_{db} + k*D_b*L_{sh}]/Cos(Ha)}$$

where

$\boldsymbol{A_C}$ **= Contact Area in square inches**

k = Land-Width Ratio = 0.30 here

$\boldsymbol{D_g}$ **= Groove ID of Barrel = 0.3380 inches**

$\boldsymbol{D_b}$ **= Bore ID of Barrel = 0.3300 inches**

$\boldsymbol{L_{db}}$ **= Top Length of Rear Driving Band (RDB) = 0.2334 in.**

$\boldsymbol{L_{sh}}$ **= Total Shank Length (including RDB) = 0.6934 inches**

Ha = Helix Angle of Rifling = $\boldsymbol{Tan^{-1}[\pi/n]}$ **= 9.81 degrees**

n = 18.17 = Number of Calibers/Turn of the Rifling.

Thus, the total contact area $\boldsymbol{A_C}$ for this example is

$$\boldsymbol{A_C = 0.3949 \text{ square inches.}}$$

One could argue that including the cosine projection at the helix angle **Ha** into the engraving length is neither necessary nor correct, but it is a small effect in either case (**1.5 percent**), even for this rather steep helix angle **Ha**. Actually, the ***widths*** of the engraving marks are *reduced* by this same cosine projection effect at the helix angle **Ha**, so the contact area A_C truly is independent of this helix angle.

We merely wished to show that, contrary to the prevailing perception, the selection of any reasonable rifling twist-rate actually has ***no effect*** on shot-start pressure P_0. With an axial second moment of inertia **Ix** of just **2.9075 grain-inches²**, the initial spin-up torque for this copper bullet requires *negligible* axial force even at this rather steep helix angle of **9.81 degrees**, especially after taking into account its **5.78:1** mechanical advantage.

Then, the total radial contact force F_C distributed over this contact area A_C at any base-pressure **P(t)** is given by

$$F_C = \sigma_r * A_C = 0.1333*P(t)*0.3949 = 0.05265*P(t) \text{ pounds}$$

This radial force F_C is a distributed normal-direction contact force in the sense ordinarily used to calculate the axial force-of-friction F_f resisting forward motion of the bullet:

$$F_f = C_f * F_C = 0.36*0.05265*P(t) = 0.01895*P(t) \text{ pounds}$$

Recall the caveats mentioned earlier about bullet-to-barrel dynamic friction not being analytic.

The partial base-pressure P_f equivalent to this axial frictional force F_f acting upon the cross-sectional area of the bullet A_B is just

$$A_B = (\pi/4)*D_g^2 = 0.089727 \text{ square inches}$$

$$P_f = F_f/A_B = 0.01895*P(t)/0.089727 = 0.21125*P(t) \text{ psi}$$

In particular, when $P(t) = P_{CE} = 1038$ **psi**, as calculated in the preceding section, the total base-pressure P_0 required for shot-start is the sum of P_{CE} and P_f

$$P_f = 0.21125*1038 = 219 \text{ psi}$$

$$P_0 = P_{CE} + P_f = P_{CE} + 0.21125*P_{CE}$$

$$P_0 = 1.21125*1038 = 1257 \text{ psi.}$$

But, now the partial base-pressure needed to overcome friction has increased to

$$\mathbf{P_f = 0.21125*1257\ psi = 266\ psi}$$

and $$\mathbf{P_0 = P_{CE} + P_f = 1038 + 266 = 1304\ psi}$$

[etc...]

This calculation would have to be iterated several times to converge on the final result.

The actual shot-start pressure $\mathbf{P_0}$ must be slightly greater than that first-iteration calculated pressure because we have a mechanical system here incorporating fractional positive-feedback. The base-pressure $\mathbf{P_f}$ needed to overcome bullet friction both depends upon **P(t)** itself and is additively combined back with **P(t)** to form the shot-start pressure $\mathbf{P_0}$. That is to say, any friction at all causes compounded additional friction here.

We recognize this as a positive fractional-feedback, closed-loop system with unit amplification $\boldsymbol{\alpha = 1.0}$ and feedback gain $\boldsymbol{\beta = 0.21125 < 1.0}$. The loop-gain $\boldsymbol{\gamma}$ for such a positive-feedback system when $\boldsymbol{\alpha*\beta < 1.0}$ is given by

$$\boldsymbol{\gamma = \alpha/(1 - \alpha*\beta) = 1/(1 - 0.21125) = 1.26783}$$

So, now we can calculate in closed form the total shot-start pressure $\mathbf{P_0}$, including friction, as

$$\mathbf{P_0 = \boldsymbol{\gamma}*P_{CE} = 1.26783*1038 = \underline{1316\ psi}.}$$

This calculated shot-start pressure $\mathbf{P_0}$ does not include the pressure required to push the bullet from the neck of the cartridge case, which must have occurred prior to bullet engraving with these dual-diameter copper bullets. Unless our chamber has no ball seat, these new copper bullets cannot be loaded with the rear driving band pushed up solid against the origin of the rifling, as in “jam seating,” which would increase shot-start pressure by at least whatever pressure is required to push the bullet from its case neck.

This calculated shot-start pressure of **1316 psi** for these monolithic copper ULD bullets is only **36.3 percent** of the default “shot-start” pressure value of **3626 psi (25 MPa)** used in the interior ballistics program QuickLOAD©

for soft match-style jacketed, lead-cored rifle bullets, such as our comparative example 250-grain Sierra MatchKing bullet.

This **1316 psi** shot-start pressure is more in line with the QL-recommended value of **1800 psi**, including the pressure required for pushing the bullet from the case neck, when using monolithic soft brass bullets. A shot-start pressure of **6525 psi** is recommended in QL when using hard military or hunting style jacketed, lead-alloy-cored bullets or when using more conventionally designed monolithic copper bullets.

Both the peak chamber pressure and the muzzle velocity increase somewhat with any increase in shot-start pressure, and vice-versa, so this very low estimated shot-start pressure might allow slightly larger powder charges or marginally faster burning-rate propellants to be used when firing these light-weight copper bullets.

9.4 Shot-Start Pressure Summary

We found earlier that, with this same copper bullet and after considering every aspect, the net total radial contact pressure σ_r was **6.8 ksi** at a peak base-pressure **P** of **51.0 ksi** acting upon this bullet. We reasoned that this same ratio of pressures R_P

$$R_P = 6.8/51.0 = 0.21125$$

would provide a good estimation of the radial contact pressures for lower, off-peak base-pressures **P(t)** for this same bullet, barrel, and shooting conditions

$$\sigma_r(t) = R_P*\sigma_a(t) = R_P*P(t)$$

We defined the land-width fraction **k** and how it is measured at one-half the groove depth. This characteristic of the barrel's rifling pattern is a significant variable in calculating shot-start pressures.

We quantified the combined elastic compression of the copper driving band material into the grooves of the rifling, along with the plastic engravement of the rifling lands into that material, as

$$\sigma_r = (1 - k)*E*(\Delta r/r) + k*S$$

where **Δr = 0.0003 inches** is the required amount of radial compression of the rear driving band material in the rifling grooves, and **r = 0.1693 inches** here.

We also calculated the internal barrel expansion in its throat due to the radial contact pressure with the copper bullet using Lamé's Equation:

$$\mathbf{U(r_i) = 0.000281\ inches}$$

Internal barrel expansion due to these low shot-start gas pressures alone is negligible.

We formulated the reduction in copper contact pressure attributable to this radial inside barrel expansion as

$$\sigma_r = E*U(r_i)/r$$

and then formulated the net total copper-bullet contact pressure as

$$\boxed{\sigma_r(Total) = (1 - k)*E*(\Delta r/r) + k*S - E*U(r_i)/r}$$

The axial stress **σ_a(Total)** which would produce this radial stress **σ_r(Total)** is formulated as

$$\sigma_a(Total) = Tan(\Phi)*\sigma_r(Total)/\mu = P_{CE}$$

Or

$$\boxed{P_{CE} = Tan(\Phi)*\sigma_r(Total)/\mu}$$

where **Φ** is the throat half-angle, **μ** is Poisson's Ratio for copper, and **P_{CE}** is the base-pressure needed both to *compress elastically* and to *engrave plastically* the copper bullet within the expanding steel barrel, but not yet including friction considerations.

We showed that the friction between the copper bullet and the steel barrel throat constitutes a fractional positive-feedback mechanism, and how to formulate friction calculations for that type of feedback system. The base-pressure **P_f** necessary to overcome sliding friction is

$$P_f = C_f*R_P*(A_C/A_B)*P_{CE} = \beta*P_{CE}$$

where **C_f = Coefficient of Sliding Friction**

R_P = Ratio of Contact Pressure to Total Internal Pressure

A_C = Contact Area in square inches

A_B = Cross-Sectional Area of the Bore (sq. in.)

$\beta = C_f*R_P*(A_C/A_B)$ = Friction Feedback Factor.

The total shot-start pressure P_0 then initially looks like

$$P_0 = P_{CE} + P_f = P_{CE} + \beta*P_{CE} = (1 + \beta)*P_{CE}$$

But now the "friction" pressure P_f must increase because that friction now depends upon this augmented value of $P_0 > P_{CE}$ and this calculation would have to be iterated for convergence.

The closed-form expression for the gain γ for this positive-feedback friction system is

$$\gamma = 1/(1 - \beta)$$

So the shot-start pressure P_0 can be formulated in closed form as

$$\boxed{P_0 = \gamma*P_{CE} = P_{CE}/(1 - \beta) = P_{CE}/[1 - C_f*R_P*(A_C/A_B)]}$$

Note that, due to the compounding of friction here, the shot-start pressure P_0 does ***not*** vary linearly with the coefficient of dynamic friction C_f, whatever its value might be.

We found that the calculated shot-start pressure P_0 of **1316 psi** for these dual-diameter, base-drilled, monolithic copper ULD bullets fired in a very fast-twist match rifle barrel with a rather steep throat angle of **4 degrees** would be significantly less than the **3626 psi** shot-start pressure normally encountered with our comparison soft-jacketed, soft lead-cored match rifle bullet of conventional design. The calculated shot-start pressure for our new monolithic copper bullets is more in line with the **1800 psi** recommendation in QuickLOAD© for firing monolithic bullets made of soft brass.

10.0 Copper Bullet Expansion Due to Centrifugal Force

For the base-drilled, dual-diameter copper bullet, consider a thin annular copper disc (like a flat washer) of the rearmost full **0.3386-inch** diameter portion of the rear driving band, having a **0.125-inch** diameter hole drilled through its center, and rotating about the longitudinal principal axis of the bullet at **6600 revolutions per second**. This is about the fastest rotation rate ever expected with a 338-caliber projectile as it nears the muzzle of a barrel rifled at **6 inches per turn** at a speed of **3300 fps**. This disc of copper material is located within the portion of the bullet which would elastically fail first and also where the greatest radial contact pressure due to centrifugal force $\boldsymbol{\sigma_{rcf}}$ will occur.

Let us set the outside radius **Ro = 0.1693 inches**, and the inside radius **Ri = 0.0625 inches** for this annular disc. The maximum rotation rate **ω = 2π*6600 = 41,469 radians per second** as the bullet nears the muzzle.

We have Roark's formulas from mechanics giving the radial stress $\boldsymbol{\sigma_r(r)}$ and the tangential stress $\boldsymbol{\sigma_t(r)}$ for any element of this rotating disc positioned at any radial position **r** within the disc (**Ri ≤ r ≤ Ro**).

$$\sigma_r(r) = [(3+\mu)/8]*(\rho*\omega^2)*[Ro^2 + Ri^2 - r^2 - (Ro*Ri/r)^2]$$

$$\sigma_t(r) = [(3+\mu)/8]*(\rho*\omega^2)*\{Ro^2 + Ri^2 - [(1+3*\mu)/(3+\mu)]*r^2 + (Ro*Ri/r)^2\}$$

where **μ = Poisson's Ratio for Copper = 0.33**

ρ = Density of Copper = (2255.8/7000)/(12*g)

$$\rho = 8.3467*10^{-4} \text{ slugs/in}^3$$

We know that the radial stress $\boldsymbol{\sigma_r(r)}$ is **zero** at **r = Ro** and also at **r = Ri** as boundary conditions, so we do not need to evaluate $\boldsymbol{\sigma_r(r)}$ here.

We do need to evaluate the tangential stress $\boldsymbol{\sigma_t(r)}$ at **r = Ro** to see if the bullet fails elastically at this rotation rate.

$$\sigma_t(Ro) = (\rho*\omega^2/4)*[(1 - \mu)*Ro^2 + (3 + \mu)*Ri^2]$$

$$\sigma_t(Ro) = (358{,}842)*[0.0192039 + 0.0130078] = \underline{11.56 \text{ ksi}}.$$

Since $\boldsymbol{\sigma_r(Ro) = 0}$, and $\boldsymbol{\sigma_a(Ro) = -10.8}$ **ksi** (axial compression by base-pressure) at the muzzle**,** the equivalent three-dimensional von Mises stress $\boldsymbol{\sigma_{vM}}$ at **Ro** on the surface of the copper bullet leaving the muzzle is

$$\sigma_{vM}(Ro) = 0.7071*SQRT[\sigma_t^2 + \sigma_a^2 + (\sigma_t - \sigma_a)^2] = \underline{19.37 \text{ ksi}}.$$

Equivalent von Mises stress is one of several metrics used in calculating elastic failure points in mechanics when non-zero stresses occur simultaneously on any two or all three of the principal axes.

This level of von Mises stress $\boldsymbol{\sigma_{vM}(Ro)}$ is safely less than half the rated yield stress **S** of **40.0 ksi** for this half-hard copper bullet spinning at its maximum ever likely rate at the muzzle.

We can evaluate the radial displacement **U(r)** at **r = Ro** to determine the radial expansion of the spinning copper bullet as **U(Ro)**

$$\mathbf{U(Ro) = \sigma_t(Ro)*(Ro/E)}$$

$$\mathbf{U(Ro) = (11{,}560)*(1.001775*10^{-8})}$$

$$\mathbf{U(Ro) = \underline{0.0001158\ inches}.}$$

Then, the potential radial contact pressure due to centrifugal force $\boldsymbol{\sigma_{rcf}}$ as the bullet nears the muzzle would be given by

$$\mathbf{\sigma_{rcf} = E*U(Ro)/Ro = \sigma_t(Ro) = \underline{11.56\ ksi}.}$$

The copper bullet is moving at just **1272 fps** and spinning at **2544 revolutions per second** (**15,984 radians per second**) at the time of peak base-pressure **P**. At this time, the radial contact pressure is just

$$\mathbf{U(Ro) = (53{,}312)*(1.001775*10^{-8})*[0.0192039 + 0.0130078]}$$

$$\mathbf{U(Ro) = \underline{0.00001720\ inches}}$$

and $$\mathbf{\sigma_{rcf} = E*U(Ro)/Ro = \underline{1.717\ ksi}}$$

when the bullet has moved just **3.2 inches** at peak base-pressure **P**.

10.1 A Simple Approximation for Rotational Stresses

The instantaneous distributed centrifugal force **df** acting outward on a thin cylindrical shell element of mass **dm** of the rear driving/sealing band of our copper bullet or of the shank of our example jacketed bullet, at radius **r** from the spin-axis of the bullet, is given by

$$\mathbf{df = dm*r*\omega^2}$$

where $\boldsymbol{\omega}$ is the instantaneous spin-rate of the bullet in radians per second as the bullet is spinning up while traversing the rifled barrel.

The mass element **dm** of this cylindrical shell can be formulated as

$$dm = \rho*L*(2\pi*r)*dr$$

where **L** is here the axial length of the rear driving/sealing band or of the full-diameter shank of the jacketed bullet.

Combining these expressions we have

$$df = 2\pi*\rho*L*\omega^2*r^2*dr$$

Integrating over the radius **r** from **Ri = 0.0625 inches** to **Ro = 0.1693 inches** to find the total outward-acting centrifugal force **F** exerted by a 338-caliber copper bullet upon the constraining steel walls of the barrel, we have

$$F = 2\pi*\rho*L*\omega^2*[Ro^3 - Ri^3]/3$$

The tangential stress $\sigma_t = F/A$, where the distributed working area

$$A = 2\pi*r*L,$$

so $$\sigma_t = F/A = \rho*\omega^2/(3*12*g)*[(Ro^3 - Ri^3)/r]$$

The last two factors in the first denominator are necessary here only if we want to give our copper density **ρ** as a weight per unit volume **2255.8/7000 = 0.32226 pounds per cubic inch** instead of using proper density units of mass per unit volume, such as slugs per cubic inch. The acceleration of gravity **g** is taken to be **12*32.174 inches per second squared**.

Evaluating the tangential strain ratio $\varepsilon_t = \sigma_t/E$ at **Ro = 0.1693 inches** at the outer surface of the bullet, with **r = Ro**, we have

$$\varepsilon_t = 4.4813*10^{-13}*\omega^2$$

The spin rate of the bullet at the muzzle would be **6600 revolutions per second** for our example bullet fired at **3300 fps** from a barrel rifled at an extremely quick rate of **6 inches per turn**. Thus, the spin-rate **ω = 2π*6600 radians per second** at the muzzle, and the maximum strain ratio ε_t is

$$\varepsilon_t = 0.0007706$$

If unconstrained, the diameter enlargement due to centripetal force would then be **0.261 thousandths of an inch**, which is small, but not completely insignificant, occurring as a constrained radial stress near the muzzle end of the barrel and as an unconstrained bullet diameter enlargement during subsequent free flight.

Even at **6600 revolutions per second**, the radial stress due to centrifugal force $\boldsymbol{\sigma_{rcf}}$,

$$\boldsymbol{\sigma_{rcf} = 0.0007706*E = 13.0\ ksi}$$

which is only about **10 percent** larger than the more exact calculation shown above:

$$\boldsymbol{\sigma_{rcf} = \underline{11.6\ ksi}.}$$

This centrifugal enlargement in diameter of monolithic copper bullets varies with the square of bullet spin-rate, which in turn varies linearly with bullet speed down the bore. It starts at zero, has little effect at the time of peak chamber pressure **(σ_{rcf} = 1.7 ksi)**, but peaks as the bullet-speed increases near the muzzle **(σ_{rcf} = 11.6 ksi)**, just as the inertial enlargement and base-pressure ducting enlargement are reducing monotonically to their post-peak minima.

10.2 Centrifugal Expansion of Jacketed Lead-Cored Bullets

We can formulate a useful approximation of these centrifugal forces acting on our jacketed, lead-cored comparison bullet using this simplified calculation. The outward stress of the spinning lead core (with **Ri = 0**) can be calculated and added to a similar calculation of the outward stress of the spinning jacket itself. That sum can then be compared to the strength of the jacket material in preventing the bullet rupturing and disintegrating in flight.

While the jacketed bullet would certainly fail at **6600 RPS**, we can calculate a maximum bullet spin-rate and a corresponding fastest barrel twist rate which the jacketed comparison bullet might survive.

Using the simplified development above, but substituting the properties of lead for copper, we find

$$\boldsymbol{\sigma_L = \rho_L*\omega^2 *R_L{}^2/(3*12*g)\ psi}$$

with

$$\boldsymbol{\rho_L = 2867.8/7000 = 0.40969\ pounds/cu.\ in.}$$

$$\boldsymbol{R_L = 0.1692 - 0.032 = 0.1372\ inches}.$$

Then $\sigma_L = 6.65812*10^{-6} *\omega^2$ **psi**.

And for the gilding metal jacket

$$\sigma_{GM} = [\rho_{GM}*\omega^2/(3*12*g)]*[(Ro^3 - R_L^3)/Ro] \text{ psi}$$

with $\rho_{GM} = 2240.7/7000 = 0.32010$ **pounds/cu. in.**

Ro = 0.1692 inches.

Then $\sigma_{GM} = 3.69354*10^{-6} *\omega^2$ **psi**.

So, the total tangential stress on the outer part of the gilding metal jacket is

$$\sigma_{Tot} = \sigma_L + \sigma_{GM} = 1.03517*10^{-5} *\omega^2 \text{ psi}$$

But, gilding metal has a yield strength S_{GM} of only **10,000 psi** and a Modulus of Elasticity E_{GM} of **17,000,000 psi**.

If we set

$$\sigma_{Tot} = \sigma_{rcf} = S_{GM} = \underline{10.0 \text{ ksi}}$$

we can solve for the maximum spin-rate ω_{Max} which might be survived as

$$\omega_{Max} = \text{SQRT}[10{,}000/1.03517*10^{-5}] = 31{,}081 \text{ radians/second}$$

which corresponds to **<u>4947 revolutions per second (RPS)</u>**.

At a muzzle speed of **3,300 fps ,** the fastest barrel twist-rate **Tw** would be

$$Tw = 3300/4947 = 0.6671 \text{ feet/turn} = \underline{8.00 \text{ inches/turn}}.$$

The radial expansion **Δr** of the jacketed bullet at this maximum survivable spin-rate would be

$$\Delta r = R*10{,}000/17{,}000{,}000 = \underline{0.00010 \text{ inches}}.$$

<u>11.0 Conclusions</u>

A properly designed 338-caliber bullet can be CNC turned from tough, half-hard UNS C147 copper rod stock and still exceed the performance in interior ballistics of a modern soft-jacketed, soft-lead-core match bullet in every aspect except for barrel obturation contact pressure. Riflemen are well acquainted with the barrel life and bore cleaning characteristics of these fine traditional match bullets. Key design features incorporated into these 338-caliber copper ULD bullets which make this interior ballistics performance possible include (**1**) use of a dual-diameter bullet design having a bore-riding shank ahead of a wide rear driving band, (**2**) use of a **0.0006-inch** over-diameter rear driving/sealing band, and (**3**) base-drilling these 338-caliber copper bullets to a suitable depth of **0.396 inches** using a suitable drill diameter of **0.125 inches**.

With these design features incorporated, gas blow-by at peak base pressure is minimized with these copper bullets (at least compared to copper bullets of earlier designs), making for higher muzzle speeds and better uniformity of those muzzle speeds. Years of experience firing jacketed lead-cored bullets, which routinely "slug-up" to match the ID of pressure-expanded rifle barrels, did not prepare us to appreciate the difficulty which non-deforming half-hard copper bullets would encounter in sealing the peak base pressures they would encounter in rifle interior ballistics.

Even a heavy, cylindrical 338-caliber match barrel can expand internally by **1.156 thousandths of an inch** in ID at typical peak base-pressures while firing these monolithic copper bullets. Relaxing the initial elastic compression of the over-diameter rear driving band allows the half-hard copper 338 bullet to re-expand in OD by **0.6 thousandths**. The remaining **0.556 thousandths** of barrel ID expansion is less than the maximum elastic OD expansion of **0.8 thousandths** for this half-hard copper bullet. The separately analyzed elastic OD expansions of the copper bullet due to (**1**) inertial acceleration **0.336 thousandths**, (**2**) Base-drilling **0.322 thousandths**, and (**3**) centrifugal expansion **0.034 thousandths** total **0.692 thousandths of an inch**. This extra **0.136 thousandths** of bullet OD expansion capability allows the calculated **6.8 ksi** of positive internal gas-sealing radial contact pressure of the copper bullet against the steel interior of the rifle barrel.

We determined that the radial contact pressure between the outside of the bullet and the inside of the barrel at peak base-pressure was a suitable metric for comparison of gas-sealing abilities. Our copper 338-caliber ULD produces **6.8 ksi** of radial contact pressure at peak base-pressure compared to **25.4 ksi** for the 250-grain Sierra MatchKing bullet. Since our half-hard copper bullet material is about one third as strong as the barrel steel itself, care must be taken in any attempt at increasing this radial contact pressure, lest the steel barrel become overstressed in some operating conditions.

These dual-diameter copper bullets require only an estimated **35.2 percent** of the shot-start pressure expected in QuickLOAD© to be required of conventional soft-jacketed match bullets like our 250-grain Sierra MatchKing comparative example bullet here. Copper bullets of more conventional designs often encounter chamber pressure increases attributable to their much higher shot-start pressure requirements.

Friction between these copper bullets and the interior barrel steel is estimated to be only about **22.8 percent** of the friction encountered when firing conventional jacketed, lead-cored match bullets. This is mainly attributable to the much reduced area of contact for the rear driving band design of the dual-diameter copper bullets and to the much lower contact pressure of these copper bullets inside the barrel. So, barrel life and bore cleaning requirements with these new copper bullets should be no worse than what we expect with conventional bullets.

While this elementary study cannot produce firm numerical values due both to its over simplification and to the large variances to be expected among rifle bullets and barrels, at least we can see why some copper bullet designs will perform better than others within the rifle barrel.