Rifling Twist Rate Concerns

February, 2020
James A. Boatright

We have been exploiting Coning Theory in the development of our new turned copper Ultra-Low-Drag rifle bullet designs. We ran into an accuracy problem in our test-firing which we did not properly anticipate. This probably happened largely because the aeroballistic yaw attitude **α** of the coning bullet is, itself, a ***free variable*** in Coning Theory, and it cannot readily be formulated. For example, the slow-mode coning rate **ω_2** is completely independent of the coning angle **α**.

Having addressed and ameliorated in designing our copper ULD bullets many of the persistent accuracy issues experienced with conventional and monolithic rifle bullets; in-bore yawing of the engraved bullet, lateral throw-off caused by static or dynamic imbalance of jacketed bullets, gas leakage past monolithic bullets causing varying muzzle velocity losses, and a few others, we expected to see improved target accuracy in our latest test results. However, that has not yet occurred.

We also noted distressingly large variations in the times-of-flight to 1,000 yards for carefully prepared and fired shots. These times-of-flight were instrumentally measured for a 5-shot string varying only **±2 feet per second** in muzzle speed **V_0**. The average air-speed over this distance was **Mach 2.5**. Something is causing some of these bullets to exhibit significantly more air-drag than other identical bullets fired identically in the same string.

The Yaw-Drag Problem

In test-firing our prototype copper Ultra-Low-Drag rifle bullets, David Tubb measured almost **30-percent reduced mean aerodynamic drag** over 1,000 yards compared to the G7 reference projectile shape, well below the drag predictions of Bob McCoy's McDRAG estimations (and well outside his 5-percent estimation limits). We correctly attributed much of this air-drag reduction to reduced yaw-drag in those tests. David used a Schneider 338-caliber barrel with a 7.5-inch twist (**n = 22.7 calibers per turn**) which produced an initial gyroscopic stability (**Sg**) of **2.75** with our earlier copper ULD bullets. We correctly intuited that by using a rifling twist-rate of **n ≈ 20**

calibers per turn, any monolithic bullet of up to about **6 calibers** in length could fly with its lowest possible total air-drag, $\mathbf{F_D = q*S*CD_0}$.

The empirical data clearly shows that for dynamically stable rifle bullets we can significantly reduce their total air-drag by selecting faster twist-rate rifled barrels (smaller values of **n**). For dynamically stable rifle bullets, the slow-mode damping factor $\boldsymbol{\lambda_S}$ in **inverse calibers** (or $\boldsymbol{\lambda_2}$ as used here in **inverse seconds**) controls the arbitrary time **T** (in seconds) required for any initial coning angle-of-attack $\boldsymbol{\alpha_0}$ to damp down to practical insignificance, say when $\mathbf{Sin^2\alpha*CD_\alpha}$ becomes less than a thousandth ($\mathbf{10^{-3}}$) of the zero-yaw drag coefficient $\mathbf{CD_0}$.

$$\alpha(t) = \alpha_0 * exp[-\lambda_2 * t].$$

In classic linear aeroballistics, the fast-mode and slow-mode damping factors are given in ***inverse calibers of flight travel distance*** (but classically considered as being dimensionless) as

$$\lambda_F = -0.5*[H + P*(2*T - H)/SQRT(P^2 - 4*M)]$$

$$\lambda_S = -0.5*[H - P*(2*T - H)/SQRT(P^2 - 4*M)].$$

Where **H**, **P**, **T**, and **M** are standard canonical aeroballistic parameters:

$$H = [\rho*S*d/(2*m)]*\{CL_\alpha - CD - k_Y^{-2} *(CM_q + CM_{\alpha dot})\}$$

$$P = (I_X/I_Y)*\omega*d/V$$ (in radians per caliber of travel, also considered dimensionless here)

$$T = [\rho*S*d/(2*m)]*\{CL_\alpha + k_X^{-2} *CM_{p\alpha}\}$$

$$M = [\rho*S*d/(2*m)]*\{k_Y^{-2} *CM_\alpha\}$$

with $\boldsymbol{\rho*S*d/(2*m) \approx 0.0000187}$ for a 250-grain 338-caliber bullet.

Here, we are using $\boldsymbol{\omega}$ to represent the instantaneous spin-rate of the bullet in radians per second to eliminate any possible confusion arising from the several different meanings of spin-rate **p** which have been used historically. Also, the aeroballistic parameter **T** should not be confused with the arbitrary time **T** used elsewhere in this paper.

For our monolithic copper ULD bullet designs, the inverse square radii of gyrations, $\mathbf{k_X^{-2} \approx 9.0\ calibers^{-2}}$ and $\mathbf{k_Y^{-2} \approx 0.55\ calibers^{-2}}$, with $\mathbf{I_Y/I_X \approx 16.5}$.

The aeroballistic coefficients, **CD**, **CL_α**, and **CM_α** are well known to us from their use in Coning Theory. The other aeroballistic coefficients, **(CM_q + $CM_{\alpha dot}$)** and **$CM_{p\alpha}$**, are minor moments causing "pitch damping" and due to Magnus forces, respectively, and have not been used in Coning Theory.

The damping of the coning angle **α** is accomplished by the moment **$M_{\alpha dot}$** acting about the apex of the coning motion. The damping of the projectile's angle-of-attack **α** is done by a moment **M_q** acting about its CG, but always in the same directional sense and parallel to **$M_{\alpha dot}$.** Coning theory shows that the coning angle **α** is identical to the spinning projectile's aerodynamic angle-of-attack **α**. Thus, both moments are dynamically damping the same angle, but about parallel axes separated by the coning distance **D** (in feet) from the CG of the bullet to the apex of its coning motion given by:

$$D = q*S*(CL_\alpha + CD)/[m*(\omega_2)^2].$$

We can also formulate **D** in calibers as:

$$D/d = D_{APEX} = ky^2 *(R + 1)*(CL_\alpha + CD)/CM_\alpha.$$

Interestingly, the distance **D_{CP}** (in calibers) from the CG to the aerodynamic center of pressure CP is given by:

$$D_{CP} = CM_\alpha/(CL_\alpha + CD)$$

So that, $$D_{Apex}*D_{CP} = (R + 1)*ky^2 \qquad \text{(in calibers}^2\text{)}$$

with **$ky^2 \approx 1.8$ calibers^2** for our copper ULD bullets.

These two damping moments, **$M_{\alpha dot}$** and **M_q**, are difficult to measure separately, so only their ***sum*** is usually determined. All of the force and moment coefficients are empirically determined in linear aeroballistics.

Examining these classic expressions for the two damping factors, **λ_F** and **λ_S**, and having reversed the signs of the second terms from those found in some references, we note that the factors **P/SQRT[P^2 – 4M]** can be simplified by expressing that factor first in terms of the gyroscopic stability **$Sg = P^2/4M$** and then in terms of the stability ratio **$R = \omega_F/\omega_S = \omega_1/\omega_2$**, making use of the interrelationships:

$$Sg = (R + 1)^2/(4*R) > 1$$

and $$R = 2*\{Sg + SQRT[Sg*(Sg - 1)]\} - 1 > 1.$$

We often find the variable **R** to be more tractable analytically than **Sg** itself.

Dividing through by $\mathbf{P = +SQRT[P^2]}$ and substituting:

$$\mathbf{P/SQRT[P^2 - 4M] = (R + 1)/(R - 1)},$$

which is simpler and perhaps conveys more readily appreciable meaning.

Then the expressions for λ_F and λ_S (in inverse calibers) can be written as:

$$\lambda_F = -0.5*[H + (2*T - H)*(R + 1)/(R - 1)]$$

$$\lambda_S = -0.5*[H - (2*T - H)*(R + 1)/(R - 1].$$

After some algebraic manipulation, the expressions for the classic damping factors simplify to:

$$\lambda_F = [H - T*(R + 1)]/(R - 1)$$

$$\lambda_S = [-H*R + T*(R + 1)]/(R - 1).$$

Note that the algebraic signs of the terms are properly crossed and that

$$\lambda_F + \lambda_S = -H*(R - 1)/(R - 1) = -H$$

as has long been known in linear aeroballistics theory.

The stability ratio **R** must be greater than **1.00** as the lower limit for gyroscopic stability, and **R** is typically much greater than **1.00**. For example, if **R = 11.0** (**Sg = 3.273**) for a gyroscopically very stable projectile, the two classic damping factors would become

$$\lambda_F = 0.1*H - 1.2*T$$

$$\lambda_S = -1.1*H + 1.2*T$$

And, again $\quad H = -(\lambda_F + \lambda_S).$

The conditions for dynamic stability are that

$$0 < Sd = 2*T/H < 2$$

and $\quad 1/Sg < Sd*(2 - Sd).$

To continue our illustrative example, if we also set **H = (4/3)*T** (i.e., **Sd = 1.5**), these damping factors would become

$$\lambda_F = 0.133*T - 1.2*T = -1.067*T$$

$$\lambda_S = -1.466*T + 1.2*T = -0.266*T$$

which are ***dynamically stable*** fast-mode and slow-mode damping factors, since **T** is ***positive by construction*** for spin-stabilized rifle bullets.

We are instead using these two damping factors herein as

$$\lambda_1 = -\lambda_F*V/d$$

and $$\lambda_2 = -\lambda_S*V/d,$$

reversing their signs in accordance with modern engineering practice and converting them from units of ***inverse calibers of projectile travel distance*** into ***inverse seconds of flight time*** **t**, each starting with the commencement of ballistic flight.

As we shall show, the conditions for the ***dynamic stability*** of rifle bullets really only concern the slow-mode damping factor λ_S, or λ_2 as used here.

The damped coning angle **α(t)**, which is also the aerodynamic angle-of-attack **α(t)** for spin-stabilized, rotationally symmetric projectiles, can now be expressed as

$$\alpha(t) = \alpha_0*exp[-\lambda_2*t]$$

using time-of-ballistic-flight **t** as the ***independent variable***.

A benefit of this change of variables for the slow-mode damping factor is the convenient metric that whenever λ_2 equals f_2, the slow-mode coning rate in hertz, the coning angle **α(t)** will be ***critically damped*** during each coning cycle.

The *raison d'être* for this exercise is to examine how the slow-mode damping factor λ_S and, thence, our λ_2 depend upon our selection of the twist-rate **n** in calibers per turn for the rifled barrel firing the projectile.

We know from prior work that both the gyroscopic stability **Sg** and its equivalent gyroscopic stability ratio **R** (or more precisely **R + 1**) vary ***inversely with the square*** of the twist-rate **n**, or with n^{-2}:

$$R + 1 = (Ix/Iy)*\{[32\pi*m*kx^2]/[n^2*\rho*d^3*CM_\alpha]\}.$$

This handy relationship allows evaluating **R** and **Sg** whenever CM_α is known.

Examining first the expression for the more critically important slow-mode damping factor $\boldsymbol{\lambda_2}$,

$$\lambda_2 = -\lambda_S * V/d = (V/d)*[H*R/(R - 1) - T*(R + 1)/(R - 1)],$$

Substituting and again simplifying, the slow-mode damping factor becomes

$$\lambda_2 = (V/d)*[(H - T) - (2*T - H)/(R - 1)]$$

which essentially recovers the classic forms for the two aeroballistic terms.

Clearly, only the $\mathbf{(2*T - H)}$ term of this expression is principally dependent upon $\mathbf{1/R}$, since $\mathbf{1/(R - 1) = 1/R + 1/R^2 + 1/R^3 +}$.... Thus, the absolute value of this portion of the slow-mode damping factor $\boldsymbol{\lambda_2}$ is ***almost directly proportional*** to $\mathbf{n^2}$, the square of the selected rifling twist-rate **n** in calibers per turn. The aeroballistic parameter **T** is ***almost always positive***, and the $\mathbf{(2*T - H)}$ term is ***normally positive*** for spin-stabilized rifle bullets.

Since this $\mathbf{(2*T - H)}$ term usually combines ***negatively*** with the first term, ***decreasing*** its size by selecting a faster twist-rate (***smaller*** value of $\mathbf{n^2}$) normally results in a ***larger net positive*** value for $\boldsymbol{\lambda_2}$ and, thus, results in ***more rapid damping*** of the slow-mode coning motion and a ***more dynamically stable*** rifle bullet.

A similar examination of the formulation for the fast-mode damping factor yields:

$$\lambda_1 = (V/d)*[(\qquad T) + (2*T - H)/(R - 1)]$$

and

$$\lambda_2 = (V/d)*[(H - T) - (2*T - H)/(R - 1)].$$

Or, for the classic damping factors, $\boldsymbol{\lambda_F}$ and $\boldsymbol{\lambda_S}$:

$$\lambda_F = -[(\qquad T) + (2*T - H)/(R - 1)]$$

$$\lambda_S = -[(H - T) - (2*T - H)/(R - 1)].$$

These expressions for the damping factors are easier to evaluate and simpler to analyze.

Note the differences between these two damping rate expressions:

- In the first terms, **H** has disappeared for the fast mode, and the signs of **T** are reversed [**T** is almost always positive.], and

- While the second terms are identical but for their reversed signs, they are simplified, and we have nicely recovered the classic **(2*T – H)** form.

The divisor of the **(2*T – H)** term, **(R – 1) = $(\omega_1 - \omega_2)/\omega_2$**, is physically the ratio of the ***relative nutation rate*** to the ***coning rate***, since both rotations are always in the same sense. For example, if **R = 2** (or **Sg = 1.125**), the fast-mode arm makes just **R – 1 = 1** (single) rotation per coning cycle relative to the slow-mode arm, and (without considering damping) the epicyclic wind-axes plot becomes a cardioid with only one inward-pointing cusp at **t = 0**, **$2\pi/\omega_2$**, etc. As an initial condition for this case, these two arms must be equal in length and colinear. but oppositely directed, in accordance with the Law of Conservation of Angular Momentum.

As for the fast-mode damping factor, even though the **(2*T – H)** term again varies almost ***directly with*** **n^2**, the ***matching signs*** of the two terms assures that the fast mode damping factor **λ_1** will not present a dynamic stability problem for gyroscopically well stabilized rifle bullets.

Selecting a faster twist-rate barrel (smaller value of **n**), normally produces a larger slow-mode damping factor **λ_2** and thus a smaller total yaw-drag velocity retardation **ΔV**. This extra yaw-drag retardation **ΔV** starts at **t = 0** and ceases accumulating very early in ballistic flight, at some time **T** when the yaw-drag becomes arbitrarily insignificant. Obviously, this lost bullet velocity can never be recovered over the remainder of ballistic flight.

The total extra retardation **ΔV** caused by yaw-drag while any random initial yaw angle **α_0** is damping out to practical insignificance is ***inversely proportional to the square root*** of the size of **λ_2** in **inverse seconds**. We can see this by examining the extra loss in kinetic energy of the bullet **ΔKE** attributable to a small (non-zero) initial yaw attitude **α_0**:

$$\Delta KE \equiv (m/2)*(\Delta V)^2$$

and
$$\Delta KE = -q*S*CD_\alpha*V_0*\alpha_0^2 \int exp[-2*\lambda_2*t]dt$$

or
$$\Delta KE = q*S*CD_\alpha*V_0*\alpha_0^2/(2*\lambda_2).$$

So,
$$\Delta V = \alpha_0*SQRT[q*S*CD_\alpha*V_0/(m*\lambda_2)].$$

Here, we are integrating just the **δ^2** yaw-drag force component of the Drag Equation of linear aeroballistics from **t = 0** at the beginning of ballistic flight to some arbitrary later time **T** when minimum coning angle “hyper-stable” flight has been achieved. For a dynamically stable rifle bullet fired from a very quick-twist (**$n \approx 20$**) barrel, the time **T** occurs after just a very few early coning periods **T_2**, and the use of ***initial values*** is justified in this formulation.

Multiplying this yaw-drag force component by the initial velocity **V_0** yields the ***power*** initially being dissipated by this extra frictional force. Integrating this power over time (as in calculating kilowatt-hours, for example), from **0** to **T**, totals the extra ***work*** done in slowing the bullet until it achieves minimum-drag hyper-stable flight at that arbitrary later time **T**.

Hyper-stable, minimum coning angle flight is the nearest to exactly nose-forward minimum-drag flight which can be achieved by any spin-stabilized projectile fired through the air. Barring any catastrophic step-change in flight conditions, once hyper-stable flight has been achieved in early flight, the projectile continues flying with minimum air-drag throughout at least its remaining supersonic flight. The minimum coning angle in flat-firing is the change in flight path angle solely due to gravity during each subsequent coning cycle, or **$2\pi * g/(\omega_2 * V)$**.

From this expression for **ΔV** and our understanding of the approximately ***inverse square*** dependence of the slow-mode damping factor **λ_2** upon the rifling twist-rate **n**, the ***total retardation*** of the bullet over a long-distance flight **ΔV** due to (early) yaw-drag would be at least partially ***proportional to*** **n**, the rifling twist-rate in calibers per turn, and the extra retardation **ΔV** would be crudely proportional to **$\alpha_0 * n$** for any random initial yaw angle **α_0**.

We hypothesize that a randomly varying initial aeroballistic yaw, or initial yaw-rate (which has the same effect), is causing the large variation in air-drag measured in David’s tests. We attribute this unexpectedly large drag ***variation*** to random yaw, or yaw-rate, destabilization occurring while the rifle bullets are transiting the muzzle-blast region before the beginning of ballistic flight. We have added a convex radiused base onto the previously flat, square boat-tails of our copper bullets which has somewhat mitigated these initial ballistic yaw disturbances.

The spin-stabilized projectile must be considered mechanically to be a “free body” as it is about to commence ballistic flight. Thus, it can carry only its spin-rate, aeroballistic yaw, and yaw-rate attitude effects into aeroballistic flight.

As we understand the mechanics of possible random size and random orientation initial aerodynamic yaw disturbances occurring in the muzzle-blast zone, any yaw-rate picked up by the spinning projectile would be always in the same direction as (and could only increase the magnitude of) an initial yaw angle. The result would be indistinguishable from having started ballistic flight with a ***larger*** initial yaw angle, but with ***zero*** initial yaw-rate. This is analogous to the ballistician's difficulty in separating the two "pitch damping" moments mentioned earlier. The initial yaw-rate is separately damped only by $\mathbf{M_q}$, one of the two combined "pitch damping" moments. Hence, we will only consider initial yaw angles $\boldsymbol{\alpha_0}$ herein, even though a non-zero initial yaw-rate might be the primary result of muzzle-blast disturbance.

All of this being said, we should point out that the ***shortest*** measured time-of-flight over David Tubb's 1,000 yard instrumented test range indicates the individual shot which suffered the ***least*** yaw disturbance while transiting the muzzle-blast zone before commencing ballistic flight. So, its ***highest*** calculated **BC** value is the one value most representative of that bullet design's true nose-forward air-drag coefficient $\mathbf{CD_0}$. The hypothesized, random, non-zero initial yaw-drag can only slow the bullets and increase their times-of-flight. Be aware of what they actually represent before just routinely using ***statistical mean values as being most representative*** whenever a random variable has a ***single-sided distribution function*** as does $\boldsymbol{\alpha_0}$ in this case.

Aerodynamic yaw-drag is a frictional force and must be an ***even function*** in angle-of-attack $\boldsymbol{\alpha}$. In linear aeroballistics theory, the yaw-drag coefficient is pre-multiplied by

$$\delta^2 \equiv \mathrm{Sin}^2(\alpha) \approx \alpha^2.$$

with the aeroballistic yaw angle $\boldsymbol{\alpha}$ approximated for small angles as

$$\alpha \approx \mathrm{SQRT}[\mathrm{pitch}^2 + \mathrm{yaw}^2]$$

for the small aeronautical-type pitch and yaw attitudes considered here. For spin-stabilized, rotationally symmetric projectiles, there is really so such thing as a negative aerodynamic angle-of-attack $\boldsymbol{\alpha_0}$.

Larger attitude angles would require the use of sequentially ordered Eulerian attitude (gimbal) angles together with 4x4 quaternion matrix attitude transforms for any changes of coordinate systems in order to avoid mathematical ambiguity (quadrant ambiguities or singularities analogous to gimbal lock) in every case.

The Accuracy Problem

This initial random yaw-disturbance hypothesis is strongly reinforced by the disappointing accuracy results with these copper ULD bullets test-fired both by David Tubb at his 1,000-yard outdoor test range and in our wind-free 100-yard indoor test range. We each are seeing about **0.8 MOA** 5-shot groups with these fast-twist rifle barrels, even when everything else is done correctly for best accuracy. David's 338-caliber Schneider P5 test barrel is rifled at **7.5 inches** (**22.7 calibers**) per turn, while our Schneider test barrel is similarly button-rifled at **7.0 inches** (**21.2 calibers**) per turn.

The same analysis based on Coning Theory which allowed our earlier formulation of the angular trajectory deflection termed "aerodynamic jump" caused by a horizontal crosswind at the firing point holds for a rifle bullet entering a wind-free atmosphere with a non-zero initial aeroballistic yaw attitude. [In fact, this is exactly how Bob McCoy handled the simulation of firing-point crosswinds in his own 6-degree-of-freedom flight simulator.] The resulting angular deflection drives the bullet away from its intended trajectory in a radial direction 90-degrees advanced in the sense of the rifling twist from the roll orientation of the initial yaw angle itself.
As this angular deflection is given in milliradians or minutes of angle (MOA), the miss distance produced on the target is strictly proportional to firing distance (minus about 10 yards in front of the muzzle where the jump defection effectively occurs). A random magnitude initial yaw disturbance which is also randomly oriented in roll angle will simply increase "extreme spread" shot-group sizes as measured on the target.

This aerodynamic jump is caused by an impulsive aerodynamic lift-force moving the CG of the rifle bullet away from its intended trajectory during the ***first half of its first coning cycle*** during early ballistic flight. This transient lift-force is integrated over the time duration of that first ***half-period*** of the bullet's coning motion to produce a cross-track ***impulse*** (force summed over a short time interval) which shifts the direction of that bullet's linear momentum vector (without changing its magnitude) by a rectangular vector summing process. This rotating cross-track lift-force reverses its sign after ***180 degrees*** of coning motion (spanning the transient start-up event) and directionally cancels during all subsequent coning motions.

The ***size*** of this aerodynamic lift-force is ***directly proportional*** to the size of the initial random yaw angle $\mathbf{a_0}$ causing it. The amount of ***time*** over which this cross-track impulse accumulates is ***inversely proportional*** to the initial ***coning rate*** $\mathbf{f_2}$. The initial coning rate $\mathbf{f_2}$ is itself ***directly proportional*** to the firing barrel's twist-rate **n** as determined from the Tri-Cyclic Theory:

$$\mathbf{f_2 = (Ix/Iy)*[V_0/(n*d)]/(R + 1)}$$

where **Ix, Iy =** Second moments of inertia of the bullet's mass distribution about crossed principal axes
d = Caliber of the bullet in feet
$\mathbf{R = f_1/f_2 =}$ Gyroscopic stability ratio.

Both **Sg** and **R + 1** vary ***inversely with the square*** of the rifle barrel's twist-rate **n** in calibers per turn.

Examining the above expression for $\mathbf{f_2}$, the ***initial coning rate*** $\mathbf{f_2}$ in hertz varies ***directly with the rifling twist-rate*** **n**; i.e., with $\mathbf{n^{-1}/n^{-2} = n}$, which in turn causes both the cross-track impulse ***integration time*** and indeed the resulting size of that cross-track impulse to vary ***inversely*** with the value of **n**. So, the accuracy-destroying aerodynamic jump varies in size quite ***directly*** with $\mathbf{\alpha_0/n}$.

> ***"*Quicker twist-rates cause *inversely proportional* larger aerodynamic jumps."**

In examining Bob McCoy's own formulation for aerodynamic jump which is derived by calculus from the Equations of Motion, we find a factor of $\mathbf{2\pi/n}$, as its only dependence upon barrel twist-rate **n**. Both independent derivations, from the Coning Theory and from the Equations of Motion, show this same ***inverse dependence*** of the size of the aerodynamic jump upon barrel twist-rate **n**. Thus, in both independent formulations of aerodynamic jump, selecting a smaller **n** (for a faster twist-rate) ***increases*** the sizes of any aerodynamic jump trajectory deflection angles in ***inverse proportionality***.

So, in a practical sense, we could simply say that, in the presence of random initial yaw disturbances,

> "**Accuracy is *directly proportional* to the twist-rate n of the rifle barrel**."

Competitors in rifle accuracy sports have long sought to use the slowest feasible twist-rates (largest number **n**, of perhaps **40 to 60 or more calibers per turn**) in their match rifle barrels. Now we see yet another rationale supporting that acquired wisdom.

For ***best accuracy*** in the presence of some spectrum of random initial yaw disturbances, we want the slowest possible rifling twist-rate, but for ***lowest air-drag*** with the same array of initial yaw disturbances we want the much faster **20 calibers per turn** twist-rate, especially in shooting long monolithic ultra-low-drag (ULD) rifle bullets to great distances. ***We simply cannot have it both ways at once***.

In extreme long-range (ELR) riflery, we need the lowest possible air-drag to maximize supersonic range and to minimize crosswind sensitivity even at some expense in gilt-edge target accuracy. So those ELR riflemen might stick with my recommended **20 calibers per turn** twist-rates when firing monolithic ULD bullets. On the other hand, 100-yard benchrest competitors will likely stick with their **60, or more, calibers per turn** 6 mm PPC barrels. I now recommend rifling twist-rates of **24 calibers per turn** for general use with any monolithic rifle bullets. However, we have had many conventional jacketed match rifle bullets disintegrate just out of the muzzles of these fast-twist barrels.

Takeaways

The problems caused by ***yaw destabilization*** of fired rifle bullets occurring while they are transiting the muzzle-blast zone are much more serious than had been anticipated. By reducing the initial coning rate from a typical **60 to 75 hertz** for jacketed, lead-cored match bullets to the range of **25 to 45 hertz** for our copper ULD bullets fired from much faster twist-rate barrels, we have inadvertently amplified the accuracy problem by up to a **factor of**

3, both in its highly variable extra air-drag aspect and in its angular-jump accuracy destroying effects. We are currently convex-radiusing the boat-tail bases of our copper ULD bullets at **0.74-calibers**, which does help in controlling their yaw destabilization within the muzzle-blast region, and we plan to try also slightly beveling the rear corners of those boat-tails.

More research is needed into rifle building techniques which facilitate launching monolithic bullets at high speeds from very fast-twist barrels with little or no initial aeroballistic yaw or yaw-rate. Barrel porting and the use of integral suppressors come to mind, as does trying other non-tubular styles of muzzle brakes. We suspect that very high-rate gas flow through any annular ring-shaped aperture surrounding the base of the bullet within a tubular muzzle brake device might be the culprit destabilizing our bullets.

We have acquired a well proven Barrett 98B/MRAD 338-caliber muzzle brake to compare with our very effective tubular MB design. This Barrett design features two very large horizontal exit ports per side which should guarantee an extremely high gas-evacuation-rate. Perhaps artillery designers have long since gotten a handle on this yaw-destabilization problem with the high evacuation-rate brakes which they formerly attached to gun muzzles before the development of discarding sabot rounds and fin-stabilized projectiles made their continued use impractical.

With our fast-twist test barrels, we are also in a unique position to evaluate exactly how yaw-destabilizing is the use of the muzzle-attached MagnetoSpeed© type of chronograph. This type of device places a **6.5-inch** long pressure-wave reflecting planar surface along side of, and parallel to, the projected bore axis just **0.25-inch** from the near edge of the fired projectile. This is reminiscent of Dr. Franklin Mann’s “plank shooting” experiments of the late 1800’s.

Fitting high quality, very fast-twist barrels to match accurate rifles and firing very well designed and well crafted monolithic alloy rifle bullets allows us to investigate this “random initial yaw” problem more readily. Setting **n = 20 calibers per turn** for our test barrels amplifies this accuracy issue for better study.

Summary of Rifling Twist-Rate Dependence

The aeroballistic parameters which vary in magnitude ***directly with the first power of*** **n**, the rifling twist-rate given in **calibers per turn**, are:

- Coning rate (or slow-mode precession rate) $\boldsymbol{\omega_2(t)}$ in radians per second or $\mathbf{f_2(t)}$ in hertz
- Yaw-drag retardation $\mathbf{\Delta V}$ (approximately) due to initial yaw attitude $\boldsymbol{\alpha_0}$ with dynamically stable bullets.

Those parameters which vary ***inversely with the first power of*** **n** are:

- Projectile spin-rate $\boldsymbol{\omega(t)}$ of the bullet in radians per second
- Auxiliary aeroballistic parameter **P**
- Crosswind aerodynamic jump deflection angle $\mathbf{A_J}$ in MOA
- Initial-yaw-caused aerodynamic jump deflection angle $\mathbf{A_J}$
- Shot-group size increases caused by aerodynamic jump in the presence of random initial yaw attitudes
- Yaw-of-repose horizontal angle $\boldsymbol{\beta_R(t)}$ in radians
- Long-range spin-drift **SD(t)** horizontal distance in feet
- Period $\mathbf{T_2}$ of a coning cycle $\mathbf{2\pi/\omega_2(t)}$ or $\mathbf{1/f_2(t)}$ in seconds

Those parameters which vary ***inversely with the square of*** n are:

- Gyroscopic stability factor **Sg**
- Gyroscopic stability ratio **R**, as (**R + 1**) exactly
- Coning CG-to-cone-apex distance **D** in feet
- Coning radius **r = D*Sin(α)** of CG motion in feet.

Each aeroballistic coefficient is assumed to vary only with the Mach-speed of the bullet in linear aeroballistics. The mass **m** of the bullet and its spatial distribution parameters (**CG**, $\mathbf{I_X}$, $\mathbf{I_Y}$, etc.) are generally assumed to be constants. The muzzle velocity $\mathbf{V_0}$ of the fired bullet is assumed to be independent of the rifling twist-rate **n** selected for the firing barrel.